Horizontal and vertical integration in securities trading and settlement

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Abstract

This paper addresses a very European issue, the consolidation of securities trading and settlement infrastructures. In a two-country model, we analyze welfare implications of different types of consolidation. We find that in our model, full technical horizontal integration of settlement systems is better than vertical integration of exchanges and settlement systems, but vertical integration is still better than no consolidation. These findings have clear policy implications with regards to the highly fragmented European securities infrastructure.

Keywords: Securities trading and settlement, vertical and horizontal integration, substitutes and complements.


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1 Introduction

1.1 Securities trading and settlement infrastructure in Europe and the US

The European securities trading and settlement infrastructure is highly fragmented. In the 25 member states of the European Union (EU), there are about 40 securities exchanges and a similar number of central securities depositories (CSDs). In the euro area alone, i.e. in the 12 EU member states that have adopted the euro as their single currency, there are about 25 legal entities that operate regulated securities exchanges and 19 legal entities that operate a central securities depository (CSD). In the US, a single currency area of similar size, there are 12 national securities exchanges and only two CSDs!

Especially the fragmentation of the European settlement infrastructure has received a lot of attention in recent years. There is broad agreement that the high number of CSDs in the EU is an important settlement cost driver. Firstly, CSDs are characterized by relatively high fixed costs. Full technical consolidation of CSDs could therefore reduce average unit settlement costs. Secondly, financial markets in the EU are getting more and more integrated. An increasing number of securities transactions are cross-border transactions, i.e. the buyer and the seller are not located in the same country. As it is expensive to participate in several CSDs, they are typically participants of different CSDs so that the transfer of the securities from the seller to the buyer often requires a costly securities transfer across systems. Again, full technical consolidation of CSDs would reduce the volume of transfers across settlement systems and could therefore decrease settlement costs.

This argumentation is supported by comparisons of settlement costs in Europe and in the US. Lannoo and Levin (2001) for example find that the average operating income per settlement as an indicator for the settlement fee of a CSD is 1.86 times higher in the EU than the US and that the difference is to a large part due to the costs of cross-border settlement. Nera (2004) estimates that for an exchange-traded equity transaction settled on a net basis in a single CSD, the cost in the US is around $0.10 and in the EU in the range of $0.35 to $0.80 ($0.42 to $0.96), while a cross-border settlement in the EU can cost up to $35 ($42).

In the US, a full technical consolidation of all CSD (and clearing houses) for private sector securities was initiated in the 1970s and resulted in the creation of the Depository Trust & Clearing Corporation (DTCC), owner of the main US CSD, the Depository Trust Company (DTC). The other US CSD, the Fedwire Book-Entry Securities Trans-
fer System, is the CSD for public sector securities and is operated by the Federal Reserve Banks. Although the DTC is linked to the Fedwire system so that public sector securities can be transferred from one to the other CSD, a consolidation of the two CSDs has not been envisaged, i.e. the cost reduction effect of such a measure is considered to be very limited. On a national level, several European countries also technically consolidated their CSDs. In Germany for example, five CSDs were technically merged into a single CSD in 1989. Mainly due to consolidation in Italy and Spain, the number of CSDs in the euro area has gone down from 23 at the launch of the euro in January 1999 to 19 at the end of 2004.4

However, a technical integration of CSDs located in different EU countries still appears to be difficult. Legal and regulatory provisions as well as market practices create incentive to keep a technically independent CSD in each member state.5 For example, rules that stipulate that trades on a domestic securities exchange have to be settled in a domestic CSD existed in most of the EU member states and are being removed only slowly.6 As a consequence, international mergers of CSDs have so far been legal mergers without a full technical integrations of systems. Examples are Euroclear Group (Belgium, France, Netherlands, UK), Clearstream International (Germany, Luxembourg) and OMX (Finland, Latvia, Estonia). In the euro area, the number of CSDs that belong to a corporate group of CSDs has increased from zero in 1999 to ten in 2004. Most of these groups operate CSDs in more than one country and aim at a technical consolidation of the group CSDs in the long run.

However, not only consolidation of CSDs is considered as a measure to reduce transaction costs in European financial markets. Although market participants, central banks and regulators agree that consolidation is needed, there is little agreement on what kind of consolidation would be optimal. For example, it is sometimes argued against mergers of CSDs that such mergers could have an adverse effect on settlement prices as they would reduce competition between CSDs. Some people therefore prefer vertical integration, i.e. mergers of exchanges with CSDs (and clearing houses) to form so-called vertical silos. Such silos exist for example in Germany and Italy. Others favour horizontal integration of different exchanges as in Euronext Group, a corporate group that comprises the securities exchanges of France, Belgium, the Netherlands and Portugal.

In this paper, we try to shed some light on the pros and cons of the different types of consolidation in a theoretical model. We concentrate on horizontal integration of CSDs and on vertical integration and show that in our model, complete fragmentation is the worst situation from a
welfare point of view. Full technical consolidation of CSDs as it is given in the US is never disadvantageous. However, if a technical integration of CSDs is not feasible, a purely legal horizontal integration of CSDs may not always be preferable.

1.2 Towards a model of trading and settlement infrastructures

Securities exchanges and CSDs play essential roles in all major securities markets. Exchanges help to match buyers and sellers of securities. CSDs are central store houses for securities. In most countries, there is only one CSD and almost all securities issued under the country’s legislation are stored there for their entire life - as physical papers or increasingly often electronically. Furthermore, CSDs maintain records establishing ownership of securities. Major financial institutions have securities accounts with the CSD and the account balances indicate the securities owned by the respective financial institution (or its direct or indirect clients). Finally, CSDs act as major settlement service providers: they organize the transfer of securities from a seller to a buyer. If one financial institution sells securities to another, the transaction is settled by book entries in the book of the CSD: The seller’s securities account with the CSD is debited and the buyer’s securities account is credited.

Exchanges and CSDs cooperate closely. Most exchanges use for reasons of costs or for legal reasons only one CSD to settle all trades executed on the exchange. All members of the exchange have to have (directly or via an intermediary) securities accounts with that CSD. Whenever two exchange members - a seller and a buyer - are matched on the exchange, the CSD receives automatically from the exchange the instructions to debit the seller’s and to credit the buyer’s securities account. This process is called straight through processing (STP).

Special problems arise in case of cross-listed securities if the two exchanges on which the securities are listed use different CSDs for settlement. Assume that an exchange A uses only CSD A and another exchange B uses only CSD B. An investor may wish to sell on exchange B securities held on his account with CSD A. Before he can do that, the securities have to be transferred from CSD A to CSD B. For this purpose, CSDs maintain so-called (direct or indirect) links. Only after the securities have been credited to an account with CSD B, they can be sold on exchange B.

In this paper, we analyze the interactions between exchanges and CSDs in a two-country model. There is an exchange and a CSD in both countries. There are two types of securities, country A securities and country B securities. There are two types of investors, country A and
country B investors. Initially, all country A securities are held by country A investors on accounts with CSD A and all country B securities are held by country B investors with CSD B. Initially, all investors are members of their home exchange and CSD, but not of the foreign exchange and CSD. All securities are listed on both exchanges. All trades executed on exchange A must be settled in CSD A and all trades executed on exchange B must be settled in CSD B. The two CSDs maintain a link so that securities can be transferred from one CSD to the other. Country A investors want to buy B securities and country B investors want to buy A securities, i.e. due to investors’ preferences, only trades between investors from different countries are possible.

There are two ways to initiate transactions for example between a country A investor who wants to sell security A and a country B investor who wants to buy security A. Firstly, the A investor offers the securities on exchange A and the B investor orders them on exchange A. Settlement takes place in CSD A and the link is not used. This is relatively costly for the B investor who needs to become a (directly or indirectly through an intermediary) member of exchange A and CSD A. Secondly, the A investor transfers the securities through the link from CSD A to CSD B and then offers them on exchange B while the B investor orders them on exchange B. This is costly for the A investor who needs to transfer his securities through the link and must become a (direct or indirect) member of exchange B and CSD B.

Link transfers must be carried out jointly by the two CSDs. A crucial exogenous parameter of the model is the operating costs of the CSDs for providing the link service. Each CSD sets a price that the investor has to pay for this service. Furthermore, each exchange sets a price for the execution of trades and each CSD sets a price for the settlement of on-exchange trades. All four service providers are operated by profit maximizing firms.

We analyze four different industry structures: (1) Under complete separation (CS), all four service providers are operated by different independent firms and set their prices independently. (2) Under vertical integration (VI), the exchange and the CSD in both countries are operated by the same firm and thus coordinate their price setting. (3) Under horizontal integration of the CSDs (HI), both CSDs are operated by the same firm. The exchanges are operated independently. We distinguish two stages of horizontal integration: (a) Purely legal integrations (LHI): Though the CSDs are operated by the same firm, they are technologically still different systems. The transfer of securities through the link entails the same operating costs for the CSDs as under CS and VI. But the CSDs set their prices for the link transfer as well as for the settlement
of on-exchange trades in a coordinated way. (b) Technical integration (THI): Both CSDs are technologically merged into one system so that a transfer of securities from one to another CSD does not entail any operating costs so that the operating costs of the link are zero. (4) Under horizontal integration of the CSDs plus vertical integration in one country (HVI), both CSDs and the exchange in one country are operated by the same firm. The exchange in the other country is operated independently. We again distinguish two stages of horizontal integration: (a) Purely legal integration (LHVI): The two CSDs are technologically still different systems. (b) Technical integration (THVI): Both CSDs are technologically merged into one system.

Horizontal integration of CSDs may indeed always lead eventually to THI or THVI respectively. However, as mentioned in Section 1.1, there still exist barriers that prevent CSDs located in different countries from a full technical merger. Furthermore, at least in the EU, horizontal integration of CSDs across borders usually starts with purely legal integration and technical integration can be achieved only after a transition period. It may well be of interest to analyze this transition period in detail. Finally, analyzing LHI and LHVI is not redundant since it helps to distinguish two effects of the transition from CS to THI or THVI. This is a pure competition effect illustrated by the transition from CS to LHI or LHVI. And a cost reduction effect illustrated by the transition from LHI to THI or from LHVI to THVI. Any kind of merger may have these two effects. This positive cost reduction effect may however be outweighed by a negative competition effect, i.e. the fact that the merger reduces the competition in the industry. Analyzing LHI and LHVI as an intermediate step in the transition from CS to THI or THVI helps to distinguish these two effects of horizontal integration of CSDs.

A welfare comparison of the four industry structures is the center of our attention. The results of this comparison are strikingly simple: VI, LHI and LHVI entail a (weakly) higher welfare then CS. That is, the competition effects of the transition from CS to VI, to LHI and to LHVI are positive. If the link operating costs under CS, VI, LHI and LHVI exceed a certain threshold, then VI entails a higher welfare then LHVI and LHVI entails a higher welfare than LHI. Thus, the competition effect is greater in the transition to VI than in the transition to LHVI and LHI. If the operating costs of the link under CS, VI, LHI and LHVI are smaller than this threshold, then LHI and LHVI entail the same level of welfare and a higher welfare than VI, i.e. the competition effect is greater in a transition to LHI and LHVI. However, THI and THVI always entails the highest economic welfare of all structures. In other words, even if the competition effect of a transition from CS to VI is greater than the
competition effect of a transition to THI or THVI, the overall welfare improvement is still greatest in case of a transition to THI or THVI due to its cost reduction effect.

Before we explain the economic reasons for these results, it is helpful to recall a finding from basic industrial economics. Consider a standard Bertrand duopoly. In this setting, a merger of the two firms would decrease the economic welfare if the outputs of the firms are substitutes (provided that the merger does not reduce production costs). However, if the outputs are complements, then the merger would increase the welfare. The reason is the following: If two firms produce substitutes and the price of both firms is relatively high, then one firm can easily attract more demand by reducing its price a bit and boost up its profit. In equilibrium, both firms therefore set relatively low prices. If the firms instead produce (perfect) complements, then the demand at both firms depends on the sum of the prices of the two goods. Tourists for example consider the sum of the prices for the flight to a holiday destination and for the accommodation there. If the flight is cheap, they have high demand for hotel rooms even if these are relatively expensive. If now both firms set a relatively low price, one firm would not lose too much demand even if it increases its price significantly so that a higher price would result in a higher profit. In equilibrium, both firms therefore set relatively high prices. However, if now both firms merge and the new entity reduces the prices of the two complements, its profit would increase. Thus, the sum of the prices of the two complements would be lower if the two firms are vertically integrated.

Looking again on our model, we find that the exchange and the CSD of the same country offer perfect complements since trading on the exchange requires settlement in the CSD. This is why VI entails a higher welfare than CS. Now compare LHI and VI. Firstly note that (trading and) settlement in country $A$ and (trading and) settlement in country $B$ are substitutes. However, the link service provided by CSD $A$ and the link service provided by CSD $B$ are perfect complements. From that perspective, it is not immediately clear whether LHI or VI leads to a higher welfare. The reason why VI leads to a higher welfare than LHI if the link operating costs are high is simple. In this case, transferring securities through the link is too costly, i.e. the link is not used and securities are always traded where they are initially held. Thus, the CSDs in fact do not compete at all, neither in substitutes nor in complements. CS and LHI now lead exactly to the same equilibrium. However, VI leads to lower prices than CS and LHI and thus to a higher welfare because the exchange and the CSD of the same country offer perfect complements. As LHVI is a mixture of VI and LHI, it is between
VI and LHI. The reason why LHI and LHVI lead to a higher welfare than VI if the link operating costs are low is a bit more complex and will be discussed in detail later.\textsuperscript{10}

Finally, it is clear that THI leads to a higher welfare than LHI (and THVI leads to a higher welfare than LHVI) due to cost reductions. Thus, it is clear that THI and THVI are the best if the link operating costs are low since LHI and LHVI are better than CS and VI in this case. However, if the link operating costs are high, then LHI and LHVI are not better anymore than VI. But now, the cost reduction effect of THI and THVI is even more significant and THI and THVI are still better than all other industry structures.

Our findings have obvious implications for the policy discussion on what kind of consolidation may be most desirable for the European securities trading and settlement infrastructure. However, it is important to draw attention to a few important limitations of our model. Firstly, we assume that the exchanges cannot choose which CSD they use. Each exchange has to settle on the CSD located in the exchange’s country. This assumption clearly reduces the potential competition between the two CSDs significantly. If under CS, the CSDs were forced to compete with each other for the exchanges, then any type of horizontal integration of the CSDs may result in a negative competition effect. Currently, national exchanges are to a large extent bound to use exclusively the respective national CSD so that our assumption seems to be realistic enough. However, this may change at some point and it would be interesting to analyze the implications of such a change. Secondly we do not allow for OTC trading, i.e. assume that trading exclusively takes place on an exchange. Though this may appear realistic for equities, it is less realistic for bonds. Thirdly, we assume that country A investors own A securities and want to buy B securities (and vice versa). Our welfare results indeed crucially depend on this assumption. However, we will show in Section 6 that the intuition we gain from our model provides insights in the results that would be obtained under alternative types of preference.

There is a large body of literature on vertical integration. An overview is in Perry (1989). Most applications of the theory of vertical integration are on network industries in which a monopolistic upstream firm (supplier of a network) produces an essential input for several competing downstream firms (users of the network). The main issue is the implications of a vertical merger of the upstream firm with one of the downstream firms. A prominent example is Vickers (1995). However, our study largely differs from this literature in the following aspects. There is no analogy of downstream/upstream firms in a trading and set-
tlement system. Exchange do not receive services from and do not make payments to CSDs and the other way round. Furthermore, there is only one upstream firm in network industries while we have two exchanges and two CSDs in our model.

Another body of literature looks at competition among interconnected networks like two operator of mobile phone networks. See for examples Laffont, Rey and Tirole (1996a and 1996b) and Laffont and Tirole (1996). The situation analyzed by this literature somehow resembles the competition between two CSDs connected by a link. However, the literature on competition among interconnected networks focuses mainly on access price regulation while vertical integration is not an issue.

There is some literature on competition, cooperation and consolidation of securities exchanges. Examples are Domowitz (1995) and Shy and Tarkka (2001). A general discussion of competition among exchanges with a special view to Europe is Di Noia (1998). In this type of literature, exchanges are often considered as networks and consolidation of different networks as a way to pool liquidity in one place. There is currently very little literature on competition and consolidation of different settlement service providers. Examples are the empirical paper by Schmiedel, Malkamaki and Tarkka (2002), the theoretical work of Holthausen and Tapking (2004), Kauko (2003) and Koeppl and Monnet (2004). Kauko (2003) is related to our paper in that it looks at links as devices that create competition between CSDs. Koeppl and Monnet (2004) is to our knowledge the only other paper that looks at exchanges and CSDs in one model. They show in a two-country model with an exchange and a CSD in each country that due to asymmetric information, an efficient merger of the CSDs cannot always be achieve if initially the exchange and the CSD are integrated in both countries. Note that these authors analyze a setting somewhat similar to ours, but look at a different question. They show that market forces may not always result in a merger of the two CSDs even if, by assumption, such a merger is welfare improving. We however ask in our paper under which conditions a merger of the two CSDs is welfare improving. We do not discuss whether under these conditions, market forces would indeed result in a merger.

The paper is organized as follows: Section 2 describes the assumption of our model. Since our model is a two-stage model, we analyze the two stages in turn in the sections 3 and 4. Section 3 looks at the behavior of investors for given prices of the exchanges and CSDs. Section 4 analyses the price setting behavior of the exchanges and CSDs under the four different industry structures described above. Finally, the welfare implications of the different industry structures are determined in section.
2 The model

There are two countries $A$ and $B$. There is a stock exchange in each country. Furthermore, there is a CSD in both countries. All trades on the stock exchange of country $A$ are settled in the CSD of country $A$ and all trades on the stock exchange of country $B$ are settled in the CSD of country $B$. For each trade executed on exchange $A$ and then settled in CSD $A$, both the seller and the buyer have to pay $p_T^A$ to stock exchange $A$ and $p_S^A$ to CSD $A$. Similarly, the prices in country $B$ are $p_T^B$ and $p_S^B$. The exchanges’ marginal costs for executing a trade are $c_T$ and the CSDs’ marginal costs for settling a trade executed on an exchange are $c_S$.

The two CSDs maintain a bilateral (direct) link which can be used to transfer securities from one CSD to the other. For each transfer, CSD $A$ charges a price $q_A$ and CSD $B$ charges $q_B$. Each CSD incurs marginal costs of $c_L$ for such transfers.

In each country, there is a set $[0,1]$ of investors. Each country $A$ investor is a member of stock exchange $A$ and of CSD $A$ (i.e. has a securities account in CSD $A$). Similarly, each country $B$ investor is a member of stock exchange $B$ and of CSD $B$ (i.e. has a securities account in CSD $B$). Initially, no country $A$ investor is a member of exchange $B$ or CSD $B$ and no country $B$ investor is a member of exchange $A$ or CSD $A$. However, any investor can decide to become a (direct or indirect) member of the foreign stock exchange and CSD to be able to trade there. For simplicity, we assume that the exchanges and CSDs do not ask for a fee for this remote access. However, the (exogenous) costs for establishing remote access - $t_T$ for access to the respective exchange and $t_S$ for access to the respective CSD - are borne by the investor. These costs refer to for example IT facilities an investor needs to set up and maintain to have remote access. We define $t = t_T + t_S$ as the overall costs for remote access.\textsuperscript{12}

There are two stocks. Stock $A$ has been issued into CSD $A$ and stock $B$ has been issued into CSD $B$. Both stocks are listed (and quoted) on both exchanges and thus eligible for settlement in both CSDs. Initially, each country $A$ investor owns one share of stock $A$ which is kept on his account with CSD $A$. Each country $B$ investor owns one share of stock $B$ which is kept on his account with CSD $B$. Country $A$ investors can sell stock $A$ and buy stock $B$ and country $B$ investors can sell stock $B$ and buy stock $A$. However, buying or selling abroad requires that the investor first becomes a member of the foreign exchange and CSD. Moreover, selling abroad requires that the investor first transfers his
share to the foreign CSD.

An investor's benefit of holding stocks depends on the location of the investor: For A investor \( i \in [0,1] \), the benefit of holding one share of stock A is \( i \) and the benefit of holding one share of stock B is \( 1 - i \). For B investor \( i \in [0,1] \), the benefit of holding on share of stock B is \( i \) and the benefit of holding one share of stock A is \( 1 - i \). Each investor has nine alternatives given by the following table:

<table>
<thead>
<tr>
<th>Alternative 1</th>
<th>Do not trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternative 2</td>
<td>Sell home stock at home, do not trade foreign stock</td>
</tr>
<tr>
<td>Alternative 3</td>
<td>Sell home stock abroad, do not trade foreign stock</td>
</tr>
<tr>
<td>Alternative 4</td>
<td>Do not trade home stock, buy foreign stock at home</td>
</tr>
<tr>
<td>Alternative 5</td>
<td>Do not trade home stock, buy foreign stock abroad</td>
</tr>
<tr>
<td>Alternative 6</td>
<td>Sell home stock at home, buy foreign stock at home</td>
</tr>
<tr>
<td>Alternative 7</td>
<td>Sell home stock at home, buy foreign stock abroad</td>
</tr>
<tr>
<td>Alternative 8</td>
<td>Sell home stock abroad, buy foreign stock at home</td>
</tr>
<tr>
<td>Alternative 9</td>
<td>Sell home stock abroad, buy foreign stock abroad</td>
</tr>
</tbody>
</table>

Let \( p_A \equiv p_A^T + p_A^S \), \( p_B \equiv p_B^T + p_B^S \) and \( q \equiv q_A + q_B \). For country A investor \( i \in [0,1] \), the benefits from each alternative are given by the following table:

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( u_{i,A}^1 = i )</td>
<td></td>
</tr>
<tr>
<td>2 ( u_{i,A}^2 = r_{AA} - p_A )</td>
<td></td>
</tr>
<tr>
<td>3 ( u_{i,A}^3 = r_{AB} - t - q - p_B )</td>
<td></td>
</tr>
<tr>
<td>4 ( u_{i,A}^4 = i + 1 - i - r_{BA} - p_A )</td>
<td></td>
</tr>
<tr>
<td>5 ( u_{i,A}^5 = i + 1 - i - r_{BB} - t - p_B )</td>
<td></td>
</tr>
<tr>
<td>6 ( u_{i,A}^6 = r_{AA} + 1 - i - r_{BA} - 2p_A )</td>
<td></td>
</tr>
<tr>
<td>7 ( u_{i,A}^7 = r_{AA} + 1 - i - r_{BB} - t - p_A - p_B )</td>
<td></td>
</tr>
<tr>
<td>8 ( u_{i,A}^8 = r_{AB} + 1 - i - r_{BA} - t - q - p_A - p_B )</td>
<td></td>
</tr>
<tr>
<td>9 ( u_{i,A}^9 = r_{AB} + 1 - i - r_{BB} - t - q - 2p_B )</td>
<td></td>
</tr>
</tbody>
</table>

Whenever we use in this paper symbols with subscripts consisting of two letters, each either A or B (e.g. \( r_{AA}, x_{BA}, s_{BB} \)), then the first letter refers to a stock and the second to a country. Here, \( r_{AA} \) is the price of stock A in country A, \( r_{BA} \) is the price of stock B in country A and \( r_{BB} \) denote the respective prices in country B. If we replace in the table the index A by B and the index B by A wherever A and B occur, then we get the benefits of country B investor \( i \in [0,1] \) for each of his nine alternatives. Note that there are economies of scope in international trading. As long as \( p_A = p_B, r_{AA} \leq r_{AB} \) and \( r_{BB} \leq r_{BA} \), an investor who has made the investments \( t + q \) necessary to sell abroad will always buy abroad as well (\( u_{i,B}^9 > u_{i,A}^8 \)).
Decisions are taken in two steps. First, the two exchanges and the two CSDs simultaneously set the transaction prices $p^T_A, p^S_A, p^T_B, p^S_B, q_A$ and $q_B$. Second, each investor selects one alternative out of his nine alternatives to maximize his benefit given all six transaction prices and all four stock prices. Simultaneously, the four stock prices take values such that all four stock markets are in equilibrium.

Note that our model covers different arrangements for cross-border trading and settlement since we allow for direct remote access to the foreign exchange and CSD as well as indirect remote access through an intermediary. However, it is assumed that all trades are executed on an exchange. In other words, we assume that there is no international custodian bank that could internalize international buy and sell orders without routing them to an exchange.

3 Stock market equilibrium

In this section, we discuss the stock market equilibrium for given transaction prices. Determining the stock market equilibrium is quite a cumbersome exercise. Firstly, for given stock prices $r_{AA}, r_{BA}, r_{BB}$ and $r_{AB}$ and given transaction prices $p^T_A, p^S_A, p^T_B, p^S_B, q_A$ and $q_B$, the demand and supply functions for stock $A$ in country $A$ ($S_{AA}$ and $D_{AA}$), for stock $A$ in country $B$ ($S_{AB}$ and $D_{AB}$), for stock $B$ in country $A$ ($S_{BA}$ and $D_{BA}$) and for stock $B$ in country $B$ ($S_{BB}$ and $D_{BB}$) have to be determined. Details can be found in Appendix A of Tapking and Yang (2004). Secondly, for given transaction prices $p^T_A, p^S_A, p^T_B, p^S_B, q_A$ and $q_B$, the stock market equilibrium has to be found. A stock market equilibrium for given transaction prices is defined as a constellation of stock prices $r_{AA}, r_{BA}, r_{BB}$ and $r_{AB}$ such that $S_{AA} = D_{AA} = x_{AA}$, $S_{AB} = D_{AB} = x_{AB}$, $S_{BA} = D_{BA} = x_{BA}$ and $S_{BB} = D_{BB} = x_{BB}$. Here, $x_{AA}, x_{AB}, x_{BA}$ and $x_{BB}$ are the equilibrium trading volumes in the four stock markets.

However, in Appendix B of Tapking and Yang (2004), it is shown that with this definition, there are under some parameter constellations multiple stock market equilibria with different trading volumes. For this reason, we apply the following refinement: If $p_A \neq p_B$, there may be two equilibria - one characterized by $x_{BB} = x_{AB} = 0$ and another by $x_{AA} = x_{BA} = 0$. In this case, we select the former if $p_A < p_B$ and the latter if $p_A > p_B$. If $p_A = p_B$, there may be infinitely many equilibria. In this case, we select the one characterized by $x_{AA} = x_{BB}$ and $x_{AB} = x_{BA}$. This refinement ensures that we have to consider exactly one equilibrium for most parameter constellations. The equilibrium is given by the following two propositions.

**Proposition 1** If $t \geq q$, then the stock market equilibrium for given
transaction prices is characterized by the following trading volumes:

I) If \( p_A < p_B \), then
\[
x_{AA} = x_{BA} = \max\{\frac{1}{2} - \frac{1}{r}(t + q) - p_A, 0\}, \quad x_{AB} = x_{BB} = 0.
\]

II) If \( p_B < p_A \), then
\[
x_{BB} = x_{AB} = \max\{\frac{1}{2} - \frac{1}{r}(t + q) - p_B, 0\}, \quad x_{BA} = x_{AA} = 0.
\]

III) If \( p_A = p_B \), then
\[
x_{AA} = x_{BA} = x_{AB} = x_{BB} = \max\{\frac{1}{4} - \frac{1}{8}(t + q) - \frac{1}{2}p_A, 0\}.
\]

If \( t \) is relatively high and \( q \) relatively low, then the link is relatively cheap and becoming a member of the exchange and the CSD of the foreign country is relatively expensive. In this case, we find that the investors are very sensitive regarding the difference between \( p_A \) and \( p_B \). As soon as these prices are not equal, all stocks are transferred via the link from the country with the higher to the country with the lower trading and settlement price and all trade takes place in the country with the lower price. I.e. trading and settlement in country A and trading and settlement in country B are perfect substitutes.

Note that if \( p_A = p_B \), all linear combinations of the trading volumes under I) and II) would characterize an equilibrium if we did not use the above refinement. Since we have \( x_{AA} = x_{BA} = x_{AB} = x_{BB} \) for \( p_A = p_B \), half of the trading country A investors sell in country B and half of the trading country B investors sell in country A. These are the investors who use the link. Furthermore, it follows that these and only these investors buy abroad. I.e. every trading investor either uses the link by himself or trades with an investor who uses the link. And those investors who do not use the link do not become members of the foreign exchange and the foreign CSD. In other words, for given trading volumes, remote membership is created as little as possible and the link is used as much as possible. In equilibrium, we find that \( r_{AB} = r_{BA} > r_{AA} = r_{BB} \), i.e. the investors who use the link are compensated by favorable stock prices for the additional transaction costs they bear. This also implies that the investors exploit the economies of scope mentioned in the previous section: No investor will ever sell abroad and buy at home.

The situation looks very different in case that \( t \leq q \):

**Proposition 2** If \( t \leq q \), then the stock market equilibrium for given transaction prices is characterized by the following trading volumes:

I) If \( |p_A - p_B| \leq \frac{1}{2}(q - t) \), then
\[
x_{AA} = x_{BA} = \max\{\frac{1-t}{2} - p_A, 0\}, \quad x_{BB} = x_{AB} = 0.
\]

II) If \( p_B - p_A \geq \frac{1}{2}(q - t) \), then
\[
x_{AA} = x_{BA} = \max\{\frac{1-t}{2} - p_A, 0\}, \quad x_{BB} = x_{AB} = \max\{\frac{1-t}{2} - p_B, 0\}, \quad x_{BA} = x_{AA} = 0.
\]

III) If \( p_A - p_B \geq \frac{1}{2}(q - t) \), then
\[
x_{BB} = x_{AB} = \max\{\frac{1-t}{2} - p_B, 0\}, \quad x_{AB} = \max\{\frac{1-t}{2} - p_A, 0\}, \quad x_{BA} = x_{AA} = 0.
\]
If $t$ is relatively low and $q$ relatively high, the link is relatively expensive and becoming a member of the foreign exchange and CSD is relatively cheap. In this case, we find that $x_{AA}$ and $x_{BB}$ are positive and $x_{AB} = x_{BA} = 0$, if the difference between $p_A$ and $p_B$ is moderate. I.e. investors from both countries become members of the respective foreign exchange and CSD and the link is not used. Stock $A$ is traded only in country $A$ and stock $B$ is traded only in country $B$. Only if the difference between $p_A$ and $p_B$ is sufficiently high, all stocks are transferred via the link from the country with the higher to the country with the lower trading and settlement price and all trade takes place in the country with the lower price. I.e. trading and settlement in country $A$ and trading and settlement in country $B$ are now imperfect substitutes.\(^{14}\)

4 Transaction price equilibrium

After determining the stock market trading volumes in equilibrium for given transaction prices, we now look at the first stage of the model. Here, the transactions prices are set by the two exchanges and the two CSDs.

4.1 Payoff functions

To begin with, we define the payoff functions of the players. The profit function of exchange $A$ is defined by

$$\pi^T_A = 2(x_{AA} + x_{BA})(p^T_A - \frac{1}{2}c_T)$$

and for exchange $B$, we get

$$\pi^T_B = 2(x_{BB} + x_{AB})(p^T_B - \frac{1}{2}c_T)$$

Note that an exchange receives the price $p^T_A$ or $p^T_B$ twice for each trade executed because both the seller and the buyer have to pay the price. This is why the trading quantities are multiplied by 2. However, $c_T$ is defined as the costs of the exchange for executing a trade.

For the CSDs, we get

$$\pi^S_A = 2(x_{AA} + x_{BA})(p^S_A - \frac{1}{2}c_S) + (x_{AB} + x_{BA})(q_A - c_L)$$

and

$$\pi^S_B = 2(x_{BB} + x_{AB})(p^S_B - \frac{1}{2}c_S) + (x_{AB} + x_{BA})(q_B - c_L)$$

The first term refers to the profits from settling trades on the respective exchange, while the second term refers to profits from operating the link.
Since we are going to look at different industry structures, we define \( \pi_A = \pi_A^T + \pi_A^S \) and \( \pi_B = \pi_B^T + \pi_B^S \) as the profit of a company operating both the exchange and the CSD in the respective country (vertical integration of exchange and CSD). The profit of a company operating both CSDs would be \( \pi_S = \pi_A^S + \pi_B^S \) (horizontal integration of the CSDs) and the profit of a company operating both CSDs and the exchange in country \( A \) would be \( \pi_A^{TS} = \pi_A^T + \pi_S \) (horizontal integration of CSDs plus vertical integration in country \( A \)).

In the following three sections, we analyze the equilibrium transaction prices and trading volumes under different industry structures. However, before we enter into the analysis, a few things should be noted.

First, for any given parameter constellation, there may be a multiplicity of no-trade equilibria which we will ignore. In case of complete separation for example, assume that both exchanges set prices that are so high that the demand for trade would be zero even if both settlement (and link) prices are zero. It is clear that any settlement prices are best responses of the operators of the CSDs. Analogously, assume that both CSDs set settlement prices that are so high that the demand for trade would be zero even if both trading prices are zero. Now any trading prices are best responses of the exchanges. Thus, there are always equilibria with no trade and prohibitively high prices for trading and settlement. Since such equilibria describe extreme coordination failures and characterize hardly interesting trading allocations, we ignore them.

Second, under certain circumstances there are equilibria in which the marginal costs for trading and/or for the settlement of trades are higher than the respective prices (i.e. \( p_A^S < \frac{1}{2} c_S \) and/or \( p_B^S < \frac{1}{2} c_S \) and \( p_A^T < \frac{1}{2} c_T \) and/or \( p_B^T < \frac{1}{2} c_T \)). We ignore these equilibria in most of the cases. Only in Subsection 4.5, we allow for them as in the industry structure analyzed there, cross-subsidization is an important feature.

Finally, to avoid corner solutions in equilibrium, we assume \( \alpha \geq 0 \) and \( \beta \geq 0 \) with

\[
\alpha \equiv 1 - t - c_S - c_T \\
\beta \equiv 1 - \frac{1}{2} t - c_S - c_T - c_L
\]

### 4.2 Complete separation (CS)

To begin with, we look at an industry in which all four service providers are operated independently by different companies. We concentrate on symmetric equilibria only, i.e. equilibria with \( p_A^S = p_B^S \), \( p_A^T = p_B^T \) and \( q_A = q_B \). The following proposition describes an equilibrium in which the link is used.
Proposition 3 If $t - 2c_L > 2\alpha$, then there is one and only one symmetric equilibrium in which the link is used.

1. If $t - 2c_L > 4\alpha$, it is characterized by
   $$\begin{align*}
p_A^T &= p_B^T = \frac{1}{2}c_T, \quad p_A^S = p_B^S = \frac{1}{2}c_S, \quad q_A = q_B = \frac{2}{3}\beta + c_L \\
x_{AA} = x_{BA} = x_{AB} = x_{BB} = \frac{1}{12}\beta
\end{align*}$$

2. If $4\alpha > t - 2c_L > 2\alpha$, then it is characterized by
   $$\begin{align*}
p_A^T &= p_B^T = \frac{1}{2}c_T, \quad p_A^S = p_B^S = \frac{1}{2}c_S, \quad q_A = q_B = \frac{1}{2}t \\
x_{AA} = x_{BA} = x_{AB} = x_{BB} = \frac{1}{4}\alpha
\end{align*}$$

If $t - 2c_L < 2\alpha$, then there is no symmetric equilibrium in which the link is used.

First note that only if the operating costs $c_L$ of the link are sufficiently low, there is an equilibrium in which the link is used. Note also that in this equilibrium, both the trading and the settlement prices are equal to marginal costs. The reason is that if $c_L$ is low, so then are the prices $q_A$ and $q_B$ for the link and it is cheap to substitute trading and settlement in one country for trading and settlement in the other country. The exchange and the CSD in country $A$ enter into perfect price competition with the exchange and CSD in country $B$. This leads to a situation in which the prices equal marginal costs. Note that part (2) of the proposition describes corner cases with $q_A = q_B = \frac{1}{2}t$.

According to proposition 3, an equilibrium in which the link is used exists only if $t - 2c_L > 2\alpha$. However, there is always an equilibrium in which the link is not used:

Proposition 4 There are always symmetric equilibria in which the link is not used. All these equilibria are characterized by

$$\begin{align*}
p_A^T &= p_B^T = \frac{1}{6}(1 - t - c_S + 2c_T), \quad p_A^S = p_B^S = \frac{1}{6}(1 - t - c_T + 2c_S), \\
q_A = q_B &\geq \frac{1}{2}t \\
x_{AA} = x_{BB} = \frac{1}{6}\alpha, \quad x_{BA} = x_{AB} = 0
\end{align*}$$

One might not be surprised to find an equilibrium in which the link is not used, if $c_L$ is high. It is also not a surprise that in such an equilibrium,
trading and settlement prices are higher than marginal costs since there is no (direct) competition between the two countries if \( c_L \) and thus the link prices are high. However, why are there equilibria in which the link is not used, if \( c_L \) is low? The reason is a simple coordination problem: If for example \( q_A \) is, say, higher than \( t \), then the link will not be used no matter how low \( q_B \) is. CSD \( B \) has therefore no reason not to set \( q_B > t \). For the same reasons, CSD \( A \) has now no reason not to set \( q_A > t \). Thus, \( q_A = q_B \geq t \) always constitutes an equilibrium.

Thus, we do not have a unique equilibrium, if \( t - 2c_L > 2\alpha \). However, there are good reasons to select in this case the equilibrium described in proposition 3. Firstly, it is easy to show that the CSDs reach a higher profit in the equilibrium described in proposition 3 than in the one of proposition 4. Secondly, we have argued in the previous subsection that we do not consider equilibria in which there is no trade at all due to coordination failures. For the same reasons, we can ignore the equilibria of proposition 4 in case that \( t - 2c_L > 2\alpha \). I.e. from now on, we assume that the CSDs coordinate on the equilibrium described in proposition 3 whenever \( t - 2c_L > 2\alpha \).

### 4.3 Vertical Integration (VI)

We now assume that in both countries the CSD and the exchange are operated by the same firm. The operator of the silo in country \( A \) sets \( p_A^S, p_A^T \) and \( q_A \), the operator of the other silo simultaneously sets \( p_B^S, p_B^T \) and \( q_B \). Again, we concentrate on symmetric equilibria only, i.e. equilibria with \( p_A^S = p_B^S, p_A^T = p_B^T \) and \( q_A = q_B \) and find:

**Proposition 5** Proposition 3 holds also under vertical integration.

The economic intuition for this result is of course the same as for proposition 3. If \( c_L \) is low, it is cheap to transfer securities from one country to the other. The operator of the exchange and the CSD in country \( A \) and the operator of those in country \( B \) enter into perfect competition so that trading and settlement fees go down to marginal costs.

As under complete separation, there is always an equilibrium in which the link is not used:

**Proposition 6** There are always symmetric equilibria in which the link
is not used. All these equilibria are characterized by

\[ p^T_A + p^S_A = p^T_B + p^S_B = \frac{1}{4}(1 - t + c_S + c_T), \]

\[ q_A = q_B \geq \frac{1}{2} t \]

\[ x_{AA} = x_{BB} = \frac{1}{4} \alpha, \quad x_{BA} = x_{AB} = 0 \]

Note that the trading and settlement prices are lower in the equilibrium of proposition 6 than in the one of proposition 4. The reason is that the CSD and the exchange of a given country offer complements. As explained in the introduction, mergers of firms that offer complements reduce prices.

Again, we do not have a unique equilibrium if \( t - 2c_L > 2\alpha \). But it is again easy to see that the CSD’s profits are higher in the equilibrium described in proposition 5 than in the one of proposition 6. For that reason and because proposition 6 describes equilibria that are due to coordination failures if \( c_L \) is low, we assume again that the CSDs coordinate on the equilibrium described in proposition 5 whenever \( t - 2c_L > 2\alpha \).

### 4.4 Horizontal integration of CSDs (HI)

We now look at a horizontally integrated structure, i.e. the two CSDs are operated by one company. In this case, we have three players. The operator of exchange \( A \) sets \( p^T_A \) and has payoff function \( \pi^T_A \). The operator of exchange \( B \) sets \( p^T_B \) and has payoff function \( \pi^T_B \). Finally, the operator of the two CSDs sets \( p^S_A, p^S_B, q_A \) and \( q_B \); his payoff function is \( \pi_S \). We assume that the operator of the CSDs cannot price-discriminate between the two countries, i.e. he has to set \( p^S_A = p^S_B \). This assumption can be justified by the existence of competition authorities like the European Commission in the EU that would not allow price discrimination. We first consider a purely legal integration of the CSDs (LHI). Concentrating only on symmetric equilibria \( (p^T_A = p^T_B) \), we get

**Proposition 7 (1)** For \( t - 2c_L > 0 \), all symmetric equilibria are characterized by

\[ p^T_A = p^T_B = \frac{1}{2} c_T, \quad q + 4p^S_A = 1 - \frac{1}{2} t + c_S + c_L - c_T, \quad p^S_A = p^S_B, \quad q \leq t \]

\[ x_{AA} = x_{BA} = x_{AB} = x_{BB} = \frac{1}{8} \beta \]
For $t - 2c_L < 0$, all symmetric equilibria are characterized by

$$p_T^A = p_T^B = \frac{1}{6} [1 - t + 2ct - cs], \ p_T^S = p_B^S = \frac{1}{6} [1 - t + 2cs - ct],$$

$$q \geq t,$$

$$x_{AA} = x_{BB} = \frac{1}{6} \alpha, \ x_{BA} = x_{AB} = 0$$

If $c_L$ is low, i.e. $t - 2c_L > 0$, then $q$ is low and the two exchanges enter into perfect price competition that leads to a situation in which the trading prices are equal to marginal costs. However, the prices for settlement and for the link are not unique, but only the weighted sum $q + 4p_A^S$ of both. The reason is subtle, but important to note. Remember that there are economies of scope in our model: if an investor decides to sell abroad, he will buy abroad as well. If he is for example a country $B$ investor, then the transaction price he has to pay to the integrated CSDs is $q + 2p_A^S$, the country $A$ investor he sells to pays $p_A^S$ and the country $A$ investor he buys from also pays $p_A^S$. The total transaction price paid by investors to the CSDs is thus $q + 4p_A^S$. If now $q$ increases and $4p_A^S$ decreases by the same amount so that $q + 4p_A^S$ remains unchanged, then the $B$ investor pays more and the two investors from country $A$ pay less. However, it can be shown that now, $r_{AB}$ increases and $r_{BB}$ decreases until the $B$ investor is compensated for that. Thus, if $q$ increases and $q + 4p_A^S$ remains unchanged, the investors’ equilibrium behavior does not change.

We still look at $t - 2c_L > 0$ and compare the prices for settlement and for the link given by the propositions 7, 3 and 5. The settlement services of the two CSDs are substitutes, while their link services are complements. For that reason, one would expect that $p_A^S$ and $p_B^S$ are higher under horizontal integration, while $q$ is higher under complete separation and under vertical integration. However, since the prices for settlement and for the link are not unique under horizontal integration, we can only compare the weighted sum $q + 4p_A^S$. It is easy to show that $q + 4p_A^S$ is higher under complete separation and under vertical integration than it is under horizontal integration. The reason is closely related to what was explained in the previous paragraph. Investors’ choice only depends on the weighted sum $q + 4p_A^S$. That reveals a relation between the two settlement services on the one hand and the two link services on the other: they are perfect complements. Using the link is beneficial for investors only if they also use the settlement service of one CSD. And the settlement service of a CSD is used only if the link services are used before. This is due to the economies of scope described above. As the settlement services and the link services are complements, prices are
lower if all these services are provided by the same player, i.e. under horizontal integration.

Finally, if \( t - 2c_L < 0 \), then the link is too expensive to be used and purely legal horizontal integration and complete separation lead to the same results.

Now consider a complete technical integration of the CSDs (THI). I.e. the two CSDs are operated on the same system and the operating costs of the link are thus \( c_L = 0 \), while \( t_S = 0 \) as well. The equilibrium for this case now follows directly from part (1) of proposition 7:

**Proposition 8** For \( c_L = t_S = 0 \) (THI), all symmetric equilibria are characterized by

\[
\begin{align*}
p^T_A &= p^T_B = \frac{1}{2} c_T, \\
q + 4p^S_A &= 1 - \frac{1}{2} t_T + c_S - c_T, \\
p^S_A &= p^S_B
\end{align*}
\]

\[
x_{AA} = x_{BA} = x_{AB} = x_{BB} = \frac{1}{8} \left[ 1 - \frac{1}{2} t_T - c_S - c_T \right]
\]

It should be mentioned here that a THI may entail substantial costs that are to be borne by the operator of the integrated CSDs. Since we ignore in our model these costs, proposition 8 describes a longer-term equilibrium while in the short run, the operator of the CSD may add a surcharge to its prices to cover the cost of THI.

### 4.5 Horizontal integration of CSDs plus vertical integration in one country (HVI)

We now consider an asymmetric industry structure. The two CSDs and the exchange in country \( A \) are operated by the same company (the integrated firm) while the exchange in country \( B \) is operated by another company. Similar to the previous subsection, we assume that the operator of the CSDs cannot price-discriminate between the two countries, i.e. has to set \( p^S_A = p^S_B \). Furthermore, we ignore possible equilibria with \( p^T_B < \frac{1}{2} c_T \) although we allow for \( p^T_A < \frac{1}{2} c_T \) as the integrated firm may use cross-subsidization technics as an important strategic tool. As we look at an asymmetric case now, we consider symmetric and asymmetric equilibria. We first consider a purely legal integration of the CSDs (LHVI).

**Proposition 9** There are always equilibria in which the link is used.\(^{16}\)

(1) For \( t - 2c_L > 0 \), all these equilibria are characterized by

\[
\begin{align*}
p^T_A &\leq p^T_B = \frac{1}{2} c_T, \\
4p_A + q &= 1 - \frac{1}{2} t + c_S + c_T + c_L, \\
p^S_A &= p^S_B, \\
q &\leq t
\end{align*}
\]
For the equilibrium with $p_T^A = p_T^B$, we get $x_{AA} = x_{BA} = x_{AB} = x_{BB} = \frac{1}{8} \beta$; For the equilibria with $p_T^A < p_T^B$, we get $x_{AA} = x_{BA} = \frac{1}{4} \beta$, $x_{AB} = x_{BB} = 0$.

(2) For $t - 2c_L < 0$, all these equilibria are characterized by

$$p_A = \frac{1}{4} (1 - t + c_S + c_T), \quad p_B - p_A \geq \frac{1}{2} (q - t), \quad p_A^S = p_B^S, \quad q = \frac{1}{2} t + c_L,$$

$$x_{AA} = \frac{1}{4} \alpha, \quad x_{BA} = \frac{1}{2} \beta - \frac{1}{4} \alpha, \quad x_{AB} = x_{BB} = 0$$

Note that for low link operating costs, i.e. $t - 2c_L > 0$, we have multiple equilibria, one is characterized by $p_T^A = p_T^B$ and the others by $p_T^A < p_T^B$. However, it is very easy to verify that the profits of both players and the social welfare are the same in all these equilibria so that we do not need to select one to continue. Also note that LHVI and LHI lead basically to the same situation if the link costs are low. The reason is simple: If the link costs are low, then the two exchanges are in perfect competition. Under LHI, the operator of the CSDs can extract all the rents as it has complete monopoly power. If we move now to LHVI, nothing changes as the operator of the CSDs, now also operator of the exchange in country $A$, still has complete monopoly power and can extract all the rents.

It is striking that we now look at an industry structure in which the link is used even if the operating costs of the link are high (i.e. $t - 2c_L < 0$). Here, the integrated firm forecloses the exchange in country $B$ completely from the market by cross-subsidizing the exchange in country $A$. It is apparent that this result cannot be socially optimal.

**Proposition 10** If and only if $t - 2c_L < -\alpha(1 - \sqrt{\frac{2}{3}})$, then there are equilibria in which the link is not used. All these equilibria are characterized by

$$p_A^T = \frac{1}{12} (1 - t + c_S + 5c_T), \quad p_B^T = \frac{1}{6} (1 - t + 2c_T - c_S),$$

$$p_B^S = p_A^S = \frac{1}{6} (1 - t + 2c_S - c_T), \quad \frac{1}{2} (q - t) \geq |p_A - p_B|$$

$$x_{AA} = \frac{1}{4} \alpha, \quad x_{BB} = \frac{1}{6} \alpha, \quad x_{AB} = x_{BA} = 0$$

Note that $x_{AA} > x_{BB}$. Again, the integrated firm forecloses the exchange in country $B$ to a certain extent from the market. However, the foreclosure is not complete as the operating costs of the link are too high to direct all trading activities to country $A$. Again, this equilibrium will turn out to be sub-optimal from a welfare point of view as prices do not reflect the underlying costs properly.
For $t - 2c_L < -\alpha(1 - \sqrt{\frac{2}{3}})$, we have multiple equilibria. However, it is easy to show that the profits of both players are greater in the equilibria of proposition 10 than in those of proposition 9 and that the equilibria described in proposition 9 are due to coordination failures. We therefore assume that the players coordinate on the equilibrium described in proposition 10, if $t - 2c_L < -\alpha(1 - \sqrt{\frac{2}{3}})$.

Similarly to the previous subsection, we now consider a complete technical integration of the CSDs (THVI). I.e. the two CSDs are operated on the same system and the operating costs of the link are thus $c_L = 0$, while $t_S = 0$ as well. The equilibrium for this case now follows directly from proposition 9:

**Proposition 11** For $c_L = t_S = 0$ (THVI), all equilibria are characterized by

$$p_T^A \leq p_B^T = \frac{1}{2}c_T, \quad 4p_A + q = 1 - \frac{1}{2}t + c_S + c_T, \quad p_A^S = p_B^S, \quad q \leq t_T$$

For the equilibrium with $p_A^T = p_B^T$, we get $x_{AA} = x_{BA} = x_{AB} = x_{BB} = \frac{1}{8}(1 - \frac{1}{2}t_T - c_T - c_S)$; For the equilibria with $p_A^T < p_B^T$, we get $x_{AA} = x_{BA} = \frac{1}{4}(1 - \frac{1}{2}t_T - c_T - c_S), \quad x_{AB} = x_{BB} = 0$.

Note that the remarks following proposition 8 apply again.

5 Welfare analysis

We now compare the welfare characteristics of the different industry structures discussed above. We first determine the general net social benefit function and describe the general welfare maximum. We than calculate the net social benefits for the different industry structures.

5.1 Net social benefit function and welfare maximum

We start with the net benefits from trade of the country $A$ investors and of the country $B$ investors. Country $A$ investors sell a volume of $x_{AA} + x_{AB}$ of stock $A$ to country $B$ investors and buy a volume of $x_{BB} + x_{BA}$ of stock $B$ from country $B$ investors. Those country $A$ investors who do not sell have a benefit of holding stock $A$ of

$$\int_{x_{AA} + x_{AB}}^{1} i \, di$$

Those country $A$ investors who sell stock $A$ have a benefit of selling it of

$$x_{AA}(r_{AA} - p_A) + x_{AB}(r_{AB} - p_B - q)$$
Those country $A$ investors who buy stock $B$ have a benefit of doing this of
\[ x_{BB} + x_{BA} \]
\[ \int_0^1 (1 - i) \, di - x_{BB}(r_{BB} + p_B) - x_{BA}(r_{BA} + p_A) \]

On top of that, country $A$ investors who trade in country $B$ must get connected to country $B$. Note the following: Each investor who wants to trade in country $B$ needs to get connected to country $B$ only once, even if he trades both stocks in country $B$. Furthermore, a situation in which some country $A$ investors trade in country $B$ only stock $A$ and some other country $A$ investors trade in country $B$ only stock $B$ is not possible. I.e. the overall number of country $A$ investors who get connected to country $B$ is \( \max\{x_{BB}, x_{AB}\} \) and the costs for country $A$ investors from getting connected to country $B$ are thus given by \( t \max\{x_{BB}, x_{AB}\} \).

The net benefit of country $A$ investors is given by the sum of these three first components minus \( t \max\{x_{BB}, x_{AB}\} \), i.e. by

\[ W_A = \frac{1}{2} - \frac{1}{2} (x_{AA} + x_{AB})^2 - \frac{1}{2} (x_{BB} + x_{BA})^2 + (x_{BB} + x_{BA}) \]
\[ -t \max\{x_{BB}, x_{AB}\} + x_{AA}(r_{AA} - p_A) + x_{AB}(r_{AB} - p_B - q) \]
\[ -x_{BA}(r_{BA} + p_A) - x_{BB}(r_{BB} + p_B) \]

We get the net benefits \( W_B \) of country $B$ investors in a similar way. The net benefits of the economy as a whole is given by

\[ W = W_A + W_B + \pi_A^T + \pi_B^T + \pi_A^S + \pi_B^S \]
\[ = (1 - c_S - c_T)[x_{AA} + x_{AB} + x_{BB} + x_{BA}] - (x_{AA} + x_{AB})^2 - (x_{BB} + x_{BA})^2 \]
\[ -2c_L(x_{AB} + x_{BA}) - t \max\{x_{BB}, x_{AB}\} - t \max\{x_{AA}, x_{BA}\} + 1 \]

It is now easy to determine the welfare maximum, i.e. the maximizer of the function \( W \). It is given by

**Proposition 12** If \( 2c_L < t \), then the welfare maximum is obtained with \( x_{AA} = x_{BB} = x_{AB} = x_{BA} = \frac{1}{4} \beta \). If \( 2c_L > t \), then it is obtained with \( x_{AA} = x_{BB} = \frac{1}{2} \alpha, x_{AB} = x_{BA} = 0 \).

Note that it is optimal to have trading of both securities in both countries and thus to use the link if the operating costs of the link are relatively low \( (2c_L < t) \). Given the relations described in the propositions 1 and 2, these welfare maximizing trading volumes can be implemented with the prices \( p_A = p_B = \frac{1}{2} c_T + \frac{1}{2} c_S \) and \( q = 2c_L \) (prices equal to marginal costs).
5.2 Comparison of social benefits for different industry structures

Taking now the results from the previous section, it is easy to calculate the net social benefits for the different industry structures. For CS, it is given by

\[ W_{CS}(cL) = \begin{cases} 
1 + \frac{5}{18} \beta^2, & \text{if } t - 2cL > 4\alpha \\
1 + \alpha \beta - \frac{1}{2} \alpha^2, & \text{if } 4\alpha > t - 2cL > 2\alpha \\
1 + \frac{5}{18} \alpha^2, & \text{if } 2\alpha > t - 2cL 
\end{cases} \]

In a similar way, we find for VI

\[ W_{VI}(cL) = \begin{cases} 
1 + \frac{5}{18} \beta^2, & \text{if } t - 2cL > 4\alpha \\
1 + \alpha \beta - \frac{1}{2} \alpha^2, & \text{if } 4\alpha > t - 2cL > 2\alpha \\
1 + \frac{3}{8} \alpha^2, & \text{if } 2\alpha > t - 2cL 
\end{cases} \]

For LHI, we get

\[ W_{LHI}(cL,t) = \begin{cases} 
1 + \frac{3}{8} \beta^2, & \text{if } t - 2cL > 0 \\
1 + \frac{3}{18} \alpha^2, & \text{if } 0 > t - 2cL 
\end{cases} \]

and for LHVI

\[ W_{LHVI}(cL,t) = \begin{cases} 
1 + \frac{3}{8} \beta^2, & \text{if } t - 2cL > 0 \\
1 + \frac{3}{8} \alpha^2 + \frac{3}{18} \beta^2 - \frac{3}{4} \alpha \beta, & \text{if } 0 > t - 2cL > -\alpha(1 - \sqrt{\frac{2}{3}}) \\
1 + \frac{47}{144} \alpha^2, & \text{if } -\alpha(1 - \sqrt{\frac{2}{3}}) > t - 2cL 
\end{cases} \]

Finally, we get from \( W_{THI} = W_{LHI}(0,t = t_T) \) and \( W_{THVI} = W_{LHVI}(0,t = t_T) \)

\[ W_{THI} = W_{THVI} = 1 + \frac{3}{8} [1 - \frac{1}{2} t_T - c_S - c_T]^2 \]

The comparison of the net social benefits is straightforward and given by

**Theorem 13** If \( t > 2cL \), then \( W_{THI} = W_{THVI} > W_{LHI} = W_{LHVI} > W_{VI} = W_{CS} \). If \( 2cL > t \), then \( W_{THI} = W_{THVI} > W_{VI} > W_{LHVI} > W_{LHI} = W_{CS} \).

To understand this result, we only have to compare equilibrium prices since it is clear that the lower the equilibrium prices the higher the social benefits. The intuition for the differences in equilibrium prices for the different industry structures has been discussed already in section 4. If \( t > 2cL \), then there is a strong complementary relation between the services provided by the two CSDs. The two link services are complements.
And the link service of one and the settlement service of the other CSD are complements. The prices are therefore lower if all these services are offered by the same player and $W_{LHI} = W_{LHV I} > W_{VI} = W_{CS}$. If $t < 2c_L$, then the link is too expensive to be used. What matters are the settlement services of the CSDs and these are neither substitutes not complements if $t < 2c_L$. Since trading and settlement within a country are complements, the prices for these services are lower if they are offered by the same player so that we get $W_{VI} > W_{LHV I} > W_{LHI} = W_{CS}$. Finally, if $2c_L > t$, then the cost saving effects of THI and THVI are so high that $W_{THI} = W_{THVI} > W_{VI}$.

6 Concluding remarks

In this paper, we have analyzed the welfare implications of different structures of the securities trading and settlement industry in a two-country model. The result of our analysis is simple: (1) A full technical integration of CSDs is always the best, no matter if there is additionally vertical integration in one of the countries (THVI) or not (THI). Thus, if mergers of CSDs easily leads to a full technical integration of the merged CSDs, then CSDs should merge. (2) If the link operating costs are low, then a purely legal integration of CSDs is the second best, again no matter if there is additionally vertical integration in one country (LHVI) or not (LHI). Thus, even if mergers of CSDs do not easily result in a full technical integration, they should still merge, provided that the link operating costs are low. (3) If the link operating costs are high, then a vertical integration in both countries is the second best. Thus, if mergers of CSDs do not easily result in a full technical integration and the link operating costs are high, the exchange and the CSD should merge in each country. Under all circumstances, complete separation is the worst.

A critical aspect of our model is the assumption that due to the preferences of investors, all trades are cross-border trades, i.e. with the buyer and seller located in different countries. However, it is now on the basis of what we have learned from our paper easier to predict what kind of results one would get under alternative assumptions. The opposite extreme would be to assume that all trades are domestic trades, i.e. between investors located in the same country. Country $A$ investors would have demand only for $A$ securities and $B$ investors only for $B$ securities. In this case, the link would never be used. Note that there would now be no competitive relation between service providers in different countries. Accordingly, horizontal integration of CSDs and complete separation would lead to exactly the same results. However, vertical integration would lead to a higher welfare than separation and horizontal integration since the only thing that matters is the fact that in each country,
the exchange and the CSD offer complements. From these consideration, a simple lesson can be learned. If domestic investors have little interest in holding foreign securities, vertical integration of domestic service providers may be desirable. If investors have instead strong preferences for foreign securities, (technical) horizontal integration of CSDs may be the best from a welfare perspective.

What however would happen in case that - say - country A investors have demand only for A securities while country B investors have demand for A and for B securities? The intuition we gain by analyzing our model again helps to understand this asymmetric case. To initiate a trade in country B between an A investor and a B would require that the A investor becomes member of the CSD and the exchange in country B and transfers his securities from CSD A to CSD B. It is obviously more efficient to initiate the trade in country A. This would only require that the country B investor becomes a member of CSD A and exchange A. One would therefore expect that all trades in A securities will be carried out in country A, all trades in B securities will be carried out in country B and the link will never be used. This suggests that the optimal industry structure might now be vertical integration in both countries.

Next, it is important to note that there is empirical evidence that CSDs are characterized by economies of scale in the sense that total settlement costs would decrease, if the number of CSDs involved decreases but the settlement volume remains constant. As indicated in the introduction, these economies of scale might be primarily due to the costs of transferring securities from one CSD to another through a link and due to high fix costs. While we assume the existence of the former in our model, we ignore the latter. What would happen if we assume that both CSDs have fix costs and that a technical merger of the two CSDs would reduce the fix costs for settlement by 1/2? Clearly, our result that full technical consolidation of the CSDs is good from a welfare point of view would still be valid as this result holds already with no fix costs as shown in this paper.

However, it should be emphasized that this paper is only a first step to analyze a complex question. More research is needed to get the complete picture and final policy conclusions should not yet be drawn. In this context, two limitations of the model mentioned already in the introduction need to be emphasized again: Firstly, we assume that the CSDs cannot compete for the exchanges, i.e. each exchange is forced to settle all trades in the domestic CSD. And secondly, we do not allow for OTC trading (including internalization of trades by an international custodian bank), i.e. all trades are executed on an exchange. Both assumptions might have influenced our results. It is left to future research
to look at the welfare implications of horizontal and vertical integration in securities trading and settlement from other angles.

Finally, it should be emphasized that we did not look at horizontal integration of exchanges. If we had done that in our model, we would probably have come to the conclusion that exchanges should not merge since they offer substitutes. However, there may still be good reasons why exchanges should merge which are not taken into account in our model. For example, mergers of exchanges lead to a concentration of liquidity. This liquidity concentration effect however does not occur in our model.

7 Appendix

Proof of propositions 1 and 2:


Proof of proposition 3:

(1) Assume \( q \leq t \) and \( p_A^S = p_B^S \). Note that under this assumption, \( p_A^T = p_B^T > \frac{1}{2}c_T \) cannot be the case in equilibrium, because each exchange could do better by slightly decreasing its price and on that way attracting all the trade. Now assume \( q \leq t \) and \( p_A^T = p_B^T \). Note that \( p_A^S = p_B^S \) implies

\[
\pi_A^S = \left[ 1 - \frac{1}{2}(t + q) - 2q_A \right] \left[ p_A^S - \frac{1}{2}c_S + \frac{1}{2}q_A - \frac{1}{2}c_L \right] = \pi_1
\]

while \( p_A^S < p_B^S \) implies

\[
\pi_A^S = \left[ 1 - \frac{1}{2}(t + q) - 2q_A \right] \left[ 2p_A^S - c_S + \frac{1}{2}q_A - \frac{1}{2}c_L \right] = \pi_2
\]

I.e. \( p_A^S = p_B^S > \frac{1}{2}c_S \) cannot be the case in equilibrium, because each CSD could do better by slightly decreasing its price and on that way attracting all the settlement of exchange trades. Thus, the following holds: If there is a symmetric equilibrium in which the link is used (and \( p_A^T = p_B^T \geq \frac{1}{2}c_T \), \( p_A^S = p_B^S \geq \frac{1}{2}c_S \)), then it is characterized by \( p_A^T = p_B^T = \frac{1}{2}c_T \), \( p_A^S = p_B^S = \frac{1}{2}c_S \).

(2) Assume \( p_A^T = p_B^T = \frac{1}{2}c_T \), \( p_A^S = p_B^S = \frac{1}{2}c_S \) as given. Maximizing \( \pi_1 \) with respect to \( q_A \) under the restriction \( q \leq t \) and then assuming \( q_A = q_B = \min \{ \frac{1}{2}t; \frac{2}{3}\beta + c_L \} \). Thus, the following holds: If there is a symmetric equilibrium in which the link is used, then it is characterized by \( q_A = q_B = \min \{ \frac{1}{2}t; \frac{2}{3}\beta + c_L \} \). Note that \( \frac{1}{2}t > \frac{2}{3}\beta + c_L \iff 4\alpha < t - 2c_L \).

(3) We finally have to check under which conditions \( p_A^T = p_B^T = \frac{1}{2}c_T \), \( p_A^S = p_B^S = \frac{1}{2}c_S \) and \( q_A = q_B = \min \{ \frac{1}{2}t; \frac{2}{3}\beta + c_L \} \) is indeed an equilibrium. It is clear that \( p_A^T = \frac{1}{2}c_T \) is a best response of exchange A on \( p_B^T = \frac{1}{2}c_T \),
$p_A^S = p_B^S$ and $q \leq t$. Now we look at CSD A and assume $p_A^T = p_B^T = \frac{1}{2} c_t$, $p_A^S = \frac{1}{2} c_S$ and $q_B = \min\{\frac{1}{2} t, \frac{2}{3} \beta + c_L\}$.

First assume $\frac{1}{2} t > \frac{2}{3} \beta + c_L$, i.e. $t - 2c_L > 4 \alpha$. This implies $q_B = \frac{2}{3} \beta + c_L$. Choosing $p_A^S = \frac{1}{2} c_S$ and $q_A = q_B$ would give $\pi_S^A = \frac{1}{6} \beta^2$. Now we show that no other strategy would give CSD A a higher profit.

(i) Alternatively, CSD A could choose a response characterized by $p_A^S < p_B^S$ and $q \leq t$. To find the best of such responses, we maximize $\pi_S^A$ under $p_A^S < p_B^S$ and $q \leq t$. This is the same as maximizing $\pi_2$ with respect to $q_A + 4p_A^S$ under $q_A + 4p_A^S \leq t - \frac{2}{3} \beta - c_L + 2c_S$. Maximizing $\pi_2$ without a constraint leads to the maximizer $q_A + 4p_A^S = \frac{2}{3} \beta + c_L + 2c_S$. This gives a profit of $\pi_S^A = \frac{1}{6} \beta^2$. Since the constrained maximization cannot lead to a higher profit, choosing a response characterized by $p_A^S < p_B^S$ and $q \leq t$ can never be better than choosing $p_A^S = \frac{1}{2} c_S$ and $q_A = q_B$.

(ii) Instead, CSD A could choose a response characterized by $p_A^S > p_B^S$ and $q \leq t$. We maximize $\pi_S^A$ under $p_A^S > p_B^S$ and $q \leq t$, which is the same as maximizing

$$\pi_S^A = \left[1 - \frac{1}{2} (t + q) - c_S - c_T\right] \left[\frac{1}{2} q_A - \frac{1}{2} c_L\right] \equiv \pi_3$$

with respect to $q_A$ under $q_A \leq t - \frac{2}{3} \beta - c_L$. Maximizing $\pi_3$ without a constraint leads to the maximizer $q_A = \frac{2}{3} \beta + c_L$. This gives a profit of $\pi_S^A = \frac{1}{6} \beta^2$. Since the constrained maximization cannot lead to a higher profit, choosing a response characterized by $p_A^S > p_B^S$ and $q \leq t$ can never be better than choosing $p_A^S = \frac{1}{2} c_S$ and $q_A = q_B$.

(iii) Furthermore, CSD A could choose a response characterized by $|p_A^S - p_B^S| \leq \frac{1}{2} (q - t)$ and $q \geq t$. Maximizing $\pi_S^A$ under $|p_A^S - p_B^S| \leq \frac{1}{2} (q - t)$ and $q \geq t$, i.e. maximizing

$$\pi_S^A = \left[1 - t - c_T - 2p_A^S\right] |p_A^S - \frac{1}{2} c_S| \equiv \pi_4$$

with respect to $p_A^S$ gives $p_A^S = \frac{1}{4} \left[1 - t + c_S - c_T\right]$ and $\pi_S^A = \frac{1}{8} \alpha^2$. It is easy to see that $\frac{1}{8} \alpha^2 < \frac{1}{6} \beta^2$ if $\frac{1}{2} t > \frac{2}{3} \beta + c_L$.

(iv) Next, CSD A could choose a response characterized by $p_A^S - p_B^S \geq \frac{1}{2} (q - t)$ and $q \geq t$. Maximizing $\pi_S^A$ under $p_A^S - p_B^S \geq \frac{1}{2} (q - t)$ and $q \geq t$, i.e. maximizing

$$\pi_S^A = \left[1 - q - c_S - c_T\right] \left[\frac{1}{2} q_A - \frac{1}{2} c_L\right] \equiv \pi_5$$

with respect to $q_A$ under $q_A \geq t - \frac{2}{3} \beta - c_L$ gives

$$q_A = \begin{cases} \frac{1}{6} \beta + \frac{1}{2} c_L + \frac{1}{4} t, & \text{if } \frac{10}{9} \beta \geq t - 2c_L \\ t - c_L - \frac{2}{3} \beta, & \text{if } \frac{10}{9} \beta \leq t - 2c_L \end{cases}$$
and

\[
\pi^A_S = \begin{cases} 
\frac{1}{2} \left[ \frac{1}{6} \beta + \frac{1}{4} (t - 2c_L)^2 \right], & \text{if } \frac{10}{9} \beta \geq t - 2c_L \\
\left[ \beta - \frac{1}{2} (t - 2c_L) \right] \left[ \frac{1}{2} (t - 2c_L) - \frac{1}{3} \beta \right], & \text{if } \frac{10}{9} \beta \leq t - 2c_L
\end{cases}
\]

It is easy to see that this is smaller than \( \frac{1}{9} \beta^2 \).

(v) Finally, CSD A could choose a response characterized by \( p^S_B - p^A_S \geq \frac{1}{2} (q - t) \) and \( q \geq t \). We have to maximize \( \pi^A_S \) under \( p^S_B - p^A_S \geq \frac{1}{2} (q - t) \) and \( q \geq t \), where

\[
\pi^A_S = [2 - t - q - 4p_A] \left[ p^A_S - \frac{1}{2} c_S \right] + \left[ 1 - q - 2p_A \right] \left[ \frac{1}{2} q_A - \frac{1}{2} c_L \right] = \pi_6
\]

Unconstrained maximization leads to \( p^A_S = \frac{1}{4} [1 - t + c_L - cT] > p^S_B = \frac{1}{2} c_S \) (and \( q_A = \frac{1}{2} t - \frac{1}{3} \beta \)), i.e. violates the constraints so that it is clear that at least one constraint must be binding. Maximizing under \( p^S_B - p^A_S = \frac{1}{2} (q - t) \) again leads to \( p^A_S = \frac{1}{4} [1 - t + c_L - cT] \), i.e. violates the constraint \( q \geq t \).

Maximizing under \( q = t \) leads to \( p^A_S = \frac{1}{4} - \frac{5}{12} t + \frac{1}{6} c_S - \frac{1}{3} c_T + \frac{1}{6} c_L \) which is smaller than \( p^A_S = \frac{1}{2} c_S \) because \( t - 2c_L > 4 \alpha \). I.e. this is the maximizer of \( \pi^A_S \) under \( p^S_B - p^A_S \geq \frac{1}{2} (q - t) \) and \( q \geq t \), and the maximum is given by \( \pi^A_S = \frac{1}{9} \beta^2 \).

This concludes the proof of part (1) of the proposition.

Now assume \( \frac{1}{3} t < \frac{1}{2} \beta + c_L \), i.e. \( t - 2c_L < 4 \alpha \). This implies \( q_B = \frac{1}{2} t \). Choosing \( p^S_A = \frac{1}{2} c_S \) and \( q_A = q_B \) would give \( \pi^A_S = \frac{1}{4} \alpha [t - 2c_L] \). Again, we show that no other strategy would give CSD A a higher profit.

(i) CSD A could alternatively choose a response characterized by \( p^S_B \leq p^A_S \) and \( q \leq t \). To find the best of such responses, we maximize \( \pi^A_S \) under \( p^A_S < p^S_B \) and \( q \leq t \), i.e. maximizing \( \pi_2 \) with respect to \( q_A + 4p^A_S \) under \( q_A + 4p^A_S \leq \frac{1}{2} t + 2c_S \). Unconstrained maximization leads to \( q_A + 4p^A_S = 1 - \frac{3}{4} t - cT + c_S + \frac{1}{2} c_L \) which is greater than \( \frac{1}{2} t + 2c_S \), because \( t - 2c_L < 4 \alpha \). I.e. the maximizer is \( q_A + 4p^A_S = \frac{1}{2} t + 2c_S \) and the maximum is \( \pi^A_S = \frac{1}{4} \alpha [t - 2c_L] \). Thus, choosing a response characterized by \( p^S_A < p_B^S \) and \( q \leq t \) can never be better than choosing \( p^S_A = \frac{1}{2} c_S \) and \( q_A = q_B \).

(ii) Next, CSD A could choose a response characterized by \( p^A_S > p^S_B \) and \( q \leq t \). We maximizing \( \pi^A_S \) under \( p^A_S > p^S_B \) and \( q \leq t \), i.e. \( \pi_3 \) with respect to \( q_A \) under \( q_A \leq \frac{1}{2} t \). Unconstrained maximization leads to \( q_A = 1 - \frac{3}{4} t - cT - c_S + \frac{1}{2} c_L \) which is greater than \( \frac{1}{2} t \), because \( t - 2c_L < 4 \alpha \). I.e. the maximizer is \( q_A = \frac{1}{2} t \) and the maximum is \( \pi^A_S = \frac{1}{4} \alpha [t - 2c_L] \). Thus, choosing a response characterized by \( p^S_A > p^S_B \) and \( q \leq t \) can never be better than choosing \( p^S_A = \frac{1}{2} c_S \) and \( q_A = q_B \).

(iii) Furthermore, CSD A could choose a response characterized by \( |p^S_A - p^S_B| \leq \frac{1}{2} (q - t) \) and \( q \geq t \). Maximizing \( \pi^A_S \) under \( |p^S_A - p^S_B| \leq \frac{1}{2} (q - t) \) and \( q \geq t \), i.e. maximizing \( \pi_4 \) with respect to \( p^S_A \) gives \( \pi^A_S = \frac{1}{8} \alpha^2 \). Since
\[
\frac{1}{6} \alpha^2 \leq \frac{1}{2} \alpha \leq t - 2c_L, \text{ we know now that } p_A^T = p_B^T = \frac{1}{2} c_T, \quad p_A^S = p_B^S = \frac{1}{2} c_S, \quad q_A = q_B = \frac{1}{2} t \text{ can be an equilibrium only if } \frac{1}{2} \alpha \leq t - 2c_L.
\]

(iv) Instead, it could choose a response characterized by \( p_A^S - p_B^S \geq \frac{1}{2} (q - t) \) and \( q \geq t \). Maximizing \( \pi_5^A \) under \( p_A^S - p_B^S \geq \frac{1}{2} (q - t) \) and \( q \geq t \), i.e. maximizing \( \pi_5 \) with respect to \( q_A \), under \( q_A \geq \frac{1}{2} t \), gives

\[
q_A = \begin{cases} 
\frac{1}{2} \alpha + \frac{1}{4} t + \frac{1}{2} c_L, & \text{if } 2 \alpha \geq t - 2c_L \\
\frac{1}{2} t, & \text{if } 2 \alpha \leq t - 2c_L
\end{cases}
\]

and

\[
\pi_5^A = \begin{cases} 
\frac{1}{4} \alpha + \frac{1}{2} t - c_L, & \text{if } 2 \alpha \geq t - 2c_L \\
\frac{1}{4} \alpha [t - 2c_L], & \text{if } 2 \alpha \leq t - 2c_L
\end{cases}
\]

It is easy to see that \( \frac{1}{4} \alpha + \frac{1}{2} t - c_L \) is greater than \( \frac{1}{4} \alpha [t - 2c_L] \), i.e. we know now that \( p_A^T = p_B^T = \frac{1}{2} c_T, p_A^S = p_B^S = \frac{1}{2} c_S, q_A = q_B = \frac{1}{2} t \) can be an equilibrium only if \( 2 \alpha \leq t - 2c_L \).

(v) Finally, CSD A could choose a response characterized by \( p_A^S - p_B^S \geq \frac{1}{2} (q - t) \) and \( q \geq t \). Now, we have to maximize \( \pi_5^A \) under \( p_B^S - p_A^S \geq \frac{1}{2} (q - t) \) and \( q \geq t \), where \( \pi_5^A = \pi_6 \). Unconstrained maximization leads to \( p_A^S = \frac{1}{2} [1 - t + c_s - c_T] \) (and \( q_A = \frac{1}{2} t + \frac{1}{2} c_L \)), i.e. to \( p_B^B = p_A^S < 0 \) so that it is clear that at least on constraint must be binding. Maximizing under \( p_B^B - p_A^S \geq \frac{1}{2} (q - t) \) leads to \( p_A^S = \frac{1}{2} [1 - t + c_s - c_T] \), i.e. to \( p_B^B - p_A^S < 0 \) so that \( p_B^S - p_A^S \geq \frac{1}{2} (q - t) \) cannot be the only binding constraint. Maximizing under \( q = t \) leads to \( p_A^S = \frac{1}{4} - \frac{5}{16} t + \frac{1}{4} c_s - \frac{1}{4} c_T + \frac{1}{4} c_L \), which is greater than \( p_B^B = \frac{1}{2} c_s \) because \( t - 2c_L < 4 \alpha \). I.e. the maximizer of \( \pi_5^A \) under \( p_B^S - p_A^S \geq \frac{1}{2} (q - t) \) and \( q \geq t \) is \( q = t \) and \( p_A^S = \frac{1}{2} c_s \). The maximum is given by \( \pi_5^A = \frac{1}{4} \alpha [t - 2c_L] \).

This concludes the proof of part (2) of the proposition.

Proof of proposition 4:

(1) Maximizing \( \pi_A^T \) under \( |p_A - p_B| \leq \frac{1}{2} (q - t) \) and assuming that \( q \geq t \) gives \( p_A^T = \frac{1}{4} [1 - t + c_T - 2p_A^T] \). Maximizing \( \pi_A^S \) under \( |p_A - p_B| \leq \frac{1}{2} (q - t) \) and \( q \geq t \) gives \( p_A^S = \frac{1}{4} [1 - t + c_S - 2p_A^S] \). Solving \( p_A^T = \frac{1}{4} [1 - t + c_T - 2p_A^T] \) and \( p_A^S = \frac{1}{4} [1 - t + c_S - 2p_A^S] \) gives \( p_A^T = \frac{1}{8} (1 - t - c_S + 2c_T), p_A^S = \frac{1}{8} (1 - t - c_T + 2c_S) \). I.e. if there is a symmetric equilibrium in which the link is not used, then it is characterized by \( p_A^T = p_B^T = \frac{1}{6} (1 - t - c_S + 2c_T) \) and \( p_A^S = p_B^S = \frac{1}{6} (1 - t - c_T + 2c_S) \). We now only need to check under which conditions this is indeed an equilibrium.

(2) Assume \( q \geq t \), \( p_B^T = \frac{1}{6} (1 - t - c_S + 2c_T) \) and \( p_A^S = p_B^S = \frac{1}{6} (1 - t - c_T + 2c_S) \). If exchange A chooses \( p_A^T = \frac{1}{6} (1 - t - c_S + 2c_T) \), it achieves a profit of \( \pi_A^T = \frac{1}{18} \alpha^2 \). Alternatively, exchange A could choose a response characterized by \( p_B - p_A \geq \frac{1}{2} (q - t) \). However, if \( q \) is sufficiently high, this cannot lead to a profit higher than \( \frac{1}{18} \alpha^2 \).
(3) Assume \( p_T^A = p_B^T = \frac{1}{6}(1 - t - c_S + 2c_T), \) \( p_S^A = \frac{1}{6}(1 - t - c_T + 2c_S) \) and some \( q_B \geq \frac{1}{2}t \). If CSD A chooses \( p_A^S = \frac{1}{6}(1 - t - c_T + 2c_S) \) and \( q_A = q_B \), then it achieves a profit of \( \pi_A^S = \frac{1}{18}\alpha^2 \). Alternatively, it could choose other responses. However, which responses are possible depends on \( q_B \). If for example \( q_B > t \), CSD A cannot choose a response that is characterized by \( q \leq t \). Furthermore, if \( q_B \) is sufficiently large, it is clear that choosing a response characterized by \( p_B - p_A \geq \frac{1}{2}(q - t) \) cannot lead to a higher profit than \( \frac{1}{18}\alpha^2 \). Similarly, if \( q_B \) is sufficiently large, it is clear that choosing a response characterized by \( p_A - p_B \geq \frac{1}{2}(q - t) \) cannot lead to a higher profit than \( \frac{1}{18}\alpha^2 \) because such a response would imply that CSD A does not settle exchange trades but makes profit only from the link which is hardly used because \( q \) is high. Thus, it is clear that there always exist a number \( f \) such that all constellations with \( p_A^T = p_B^T = \frac{1}{6}(1 - t - c_S + 2c_T), \) \( p_A^S = p_B^S = \frac{1}{6}(1 - t - c_T + 2c_S) \) and \( q_A = q_B \geq f \) are equilibria in which the link is not used.

**Proof of proposition 5:**

The proof is very similar to that of proposition 3 and therefore omitted.

**Proof of proposition 6:**

The proof is very similar to that of proposition 4 and therefore omitted.

**Proof of proposition 7:**

(1) We first determine the best response of the operator of the CSDs on \( p_A^T = p_B^T \) (under the constraint \( p_A^S = p_B^S \)):

We first maximize \( \pi_S \) under the constraint \( q \leq t \). Here, we have

\[
\pi_S = \left[ \frac{1}{2} - \frac{1}{4}t - \frac{1}{4}(q + 4p_A^S) - p_A^T \right] [q + 4p_A^S - 2c_S - 2c_L]
\]

Maximizing with respect to \( q + 4p_A^S \) gives \( q + 4p_A^S = 1 - \frac{1}{2}t + c_S + c_L - 2p_A^T \) and

\[
\pi_S = \frac{1}{4}[1 - \frac{1}{2}t - c_S - c_L - 2p_A^T]^2 \equiv \pi_S^1
\]

We now maximize \( \pi_S \) under the constraint \( q \geq t \). We have

\[
\pi_S = \left[ 1 - t - 2p_A^S - 2p_A^T \right] [2p_A^S - c_S]
\]

Maximizing with respect to \( p_A^S \) gives \( p_A^S = \frac{1}{4}[1 - t + c_S - 2p_A^T] \) and

\[
\pi_S = \frac{1}{4}[1 - t - c_S - 2p_A^T]^2 \equiv \pi_S^2
\]
Now note that \( \pi_A^1 > \pi_A^2 \iff t > 2c_L \). With that we get the following result:

If \( t > 2c_L \), we get as best responses all strategies that are characterized by \( q + 4p_A^T = 1 - \frac{1}{2}t + cs + c_L - 2p_A^T \) and \( q \leq t \).

If \( t < 2c_L \), we get as best responses all strategies that are characterized by \( p_A^S = \frac{1}{4}[1 - t + cs - 2p_A^T] \) and \( q \geq t \).

(2) We now prove part (1) of the proposition:

If \( q \leq t \), the exchanges enter into perfect Bertrand competition that leads to \( p_A^T = p_B^T = \frac{1}{2}cT \). I.e. if there is a symmetric equilibrium with trade that is characterized by \( q \leq t \), then it is characterized by \( p_A^T = p_B^T = \frac{1}{2}cT \). Assume \( t > 2c_L \). The best response of the operator of the CSDs on \( p_A^T = p_B^T = \frac{1}{2}cT \) is according to (1) given by \( q + 4p_A^T = 1 - \frac{1}{2}t + cs + c_L - ct \) and \( q \leq t \). Finally, it is easy to check that \( p_A^T = \frac{1}{2}cT \) is a best response of exchange \( A \) on \( q + 4p_A^T = 1 - \frac{1}{2}t + cs + c_L - ct \), \( q \leq t \) and \( p_B^T = \frac{1}{2}cT \).

(3) We now prove part (2) of the proposition.

Assume \( q \geq t \) and some strategies \( p_A^T \) and \( p_A^S = p_B^S \). As long as \( |p_A^T - p_B^T| \leq \frac{1}{2}(q - t) \), we have

\[
\pi_A^T = [1 - t - 2p_A^T - 2p_A^S][p_A^T - \frac{1}{2}cT] = \pi_A^{T,1}
\]

I.e. maximizing \( \pi_A^T \) and then setting \( p_A^T = p_B^T \) is the same as maximizing \( \pi_A^{T,1} \) which gives \( p_A^T = \frac{1}{4}[1 - t + ct - 2p_A^S] \). I.e. if there is a symmetric equilibrium that is characterized by \( q \geq t \), then it is characterized by \( p_A^T = p_B^T = \frac{1}{4}[1 - t + ct - 2p_A^S] \). Now assume \( t < 2c_L \). We know from (1) that if there exists a symmetric equilibrium, it is characterized by \( p_A^S = \frac{1}{4}[1 - t + cs - 2p_A^T] \) and \( q \geq t \). I.e. we know that if there is a symmetric equilibrium, it is characterized by \( p_A^T = p_B^T = \frac{1}{2} - t + 2cT + cs, p_A^S = p_B^S = \frac{1}{6} - t + 2cs - ct, q \geq t \).

To conclude the proof, we only have to show that exchange \( A \) cannot do better by deviating from \( p_A^T = \frac{1}{2} - t + 2cT + cs \), if \( p_B^T = \frac{1}{6} - t + 2cs - ct \) and \( p_A^S = p_B^S = \frac{1}{6} - t + 2cs - ct \) if \( q \geq h \) for some \( h \geq t \). However, this is immediate since the only potential better response would be to select a response characterized by \( p_B^T - p_A^T \geq \frac{1}{2}(q - t) \). But if \( q \) is sufficiently high, this would require a very small \( p_A^T \) which may even lead to a negative profit for exchange \( A \).

\[
\text{Proof of propositions 9 and 10:}
\]
The proof is cumbersome and available from the authors on request.

\[
\text{Proof of proposition 12:}
\]
The proof is very simple and therefore omitted.

32
Proof of theorem 13:
The proof is straightforward and therefore omitted.

References

[14] Laffont, Jean-Jacques, Patrick Rey, and Jean Tirole. (1998b) "Net-
Notes

1Regulated markets according to the investment services directive of the European Commission.
3See for example Schmiedel, Malkamaki and Tarkka (2002).
4See ECB (2004), p. 112.
5The only exception is Ireland that uses the British CSD Crest for Irish equities and Euroclear Belgium for Irish bonds.
6For more information, see The Giovannini Group (2001), Section 5.
7This is a strong assumption. However, in section 6, we note the effects of including investors’ demand for home securities.
8Some CSDs and exchanges are user owned. In these cases, one may argue that the assumption of profit maximization is not realistic. However, many CSDs and exchanges are organised as profit maximising entities.
9See for example Tirole (1988) and Shy (1996).
10See the two paragraphs after proposition 7 and the paragraph after proposition 9.
12If an investor is member of the foreign exchange and CSD indirectly through an intermediary, the investor would pay prices for trading and settlement abroad via its intermediary.
13We mentioned above that we select out of infinitely many equilibria the one with \( x_{AA} = x_{BB} \) and \( x_{AB} = x_{BA} \), if \( p_A = p_B \). Alternatively, we could assume that the stock market equilibrium is random and Proposition 1 (III) represents only the expected trading volumes, if \( p_A = p_B \). All our further results would remain as they are, provided that all players are risk neutral.
14Note that proposition 2 describes two equilibria for \( |p_A - p_B| = \frac{1}{2}(q - t) \). In fact, all linear combinations of these two equilibria are also equilibria in this case. We have omitted a detailed description of this case, because it has no impact on the further analysis. Furthermore, note that for \( t = q \), the two propositions describe two different equilibria.
15This is possible because companies that offer more than one service, for example settlement and link services or - in case of vertical integration - trading and settlement services, can cross-subsidize the different services.
16To avoid corner solutions, we assume \( \alpha + t - 2c_L \geq 0 \).
17Assume that LHI already leads to a cost saving effect so that it reduces the link operating costs to \( \bar{c}_L < c_L \). If \( 2\bar{c}_L \leq t \), then LHI would now outperform VI even if \( t < 2c_L \). Thus, horizontal integration is
already the best type of consolidation if it makes the economic cost of the link smaller than the cost of remote access.

18See for example Schmiedel, Malkamaki and Tarkka (2002).