Asymmetric Information, Portfolio Managers and Home Bias

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Abstract

Why do investors excessively tilt their portfolio towards domestic assets? Recent studies suggest asymmetric information plays a significant role in the home equity bias puzzle. A key assumption in theoretical models is that agents invest in assets and process information on their own. However, most international investments are executed by managers in financial institutions. These institutions spend plenty of resources on information processing, which makes the asymmetric information assumption less appealing. In this paper, we explain home bias at the fund level by showing how information asymmetry at the individual level has relevant implications at the portfolio management level. Agents delegate their investment decisions to portfolio managers of different and uncertain ability. Investors are better informed about performance of domestic markets; and therefore, are more able to evaluate the ability of managers operating in these markets. This, in turn, makes investing in domestic markets less risky and attracts more managers. Additionally, highly skilled managers benefit more from higher transparency, and this is why they are more likely to choose to operate in the domestic market. Therefore, a small information asymmetry of individual investors generates home bias due to highly skilled managers in the domestic market (higher than in the foreign market) and diversification (a higher number of managers in the domestic market). We simulate the model and find that on average 73% of investment is in the domestic market.
1 Introduction

Many explanations for the puzzles in international finance rely on asymmetric information – individual investors are believed to have more precise information about domestic markets than foreign markets.\(^1\) This is plausible since individual agents watch domestic TV, listen to domestic radio and read domestic newspapers. If agents process information and invest on their own, asymmetric information may result in home bias. However, most international investments are executed by portfolio managers in financial institutions. These managers have plenty of resources to spend on information processing, which suggests that information asymmetries at the fund level are negligible.

In this paper, we propose a model that reconciles these two observations. Even if all investment is done through mutual funds with identical access to information in all markets, asymmetric information at the individual level combined with the uncertainty about the ability of the portfolio managers may result in home bias.

The model consists of investors who delegate their investment decisions to portfolio managers of different and uncertain abilities. Each manager operates in one market only, and the entry decision is irreversible. Investors have a local monitoring advantage – they receive more information about the performance of domestic assets. When assessing the ability of the managers in the domestic market, the investors compare their performance with the domestic benchmark, while when assessing the ability of the managers operating in the foreign market, they are able to compare only the performance across managers. As a result, investors learn faster about the ability of domestic managers, which allows them to allocate their capital in the domestic market more efficiently. That, in turn, leads to home bias.

If managers are ex ante identical, investors’ bias in allocating their funds across markets attracts more managers to the domestic market. The domestic market becomes more diversified, which deepens home bias even further.

Investors are more able to assess the ability of managers operating in domestic markets; hence, in this market they are more likely to reallocate their investments to managers that are believed to be highly skilled. That implies that the domestic market is more transparent.

and it rewards the expected ability more. If ex ante investors differ in the signal about their ability, more qualified managers enter the domestic market. This induces investors to channel even more funds to the domestic market.

The home bias puzzle was raised by French and Poterba (1991) and Tesar and Werner (1995). They showed that, at the beginning of the 90’s, the fraction of stock market wealth invested domestically was around 90% for the U.S. and Japan, and around 80% for the U.K. Ahearne, Griever and Warnock (2004) updated the home bias numbers for the US and found no dramatic change. The share of domestic equity in the US portfolio in the year 2000 is around 88%, while its share in the world portfolio is 50%. Recently, Chan et al. (2005) and Hau and Rey (2008) have extensively reported the presence of home bias in the portfolios of managers in financial institutions.

We quantify our model and show that on average 73% of assets are invested in the domestic market. Although our model underestimates the empirical home bias, it comes very close given the stylized nature of it.

In addition to explaining the home bias, this model offers a testable prediction that investors obtain, on average, higher returns on their capital in domestic funds. These results are consistent with the findings on the portfolio behavior of US investors in Coval and Moskowitz (1999), Coval and Moskowitz (2001), Ivkovich and Weisbenner (2005) and Alburquerque et al. (2005).²

The paper is organized as follows. In section 2, we present a model in which all managers are ex ante identical. We show that in equilibrium there are more managers in the domestic market and there is home bias. In section 3, we extend the model by allowing the managers to differ ex ante. In section 4 we simulate the model with heterogenous managers.

2 The Model

We study a two-period economy with two countries and a continuum of investors of measure one. Investors have mean-variance preferences with a coefficient of absolute risk aversion \( \gamma \) and a unit of capital to invest in both markets. Each country has an asset market with one-period

²On the other hand, Karolyi (2002), Grinblatt and Keloharju (2000) Seasholes (2004), find that in some countries, foreign investors outperform local investors. Although not inconsistent, our paper does not offer any insight on this phenomenon.
claims. The domestic market offers a benchmark with return $v^D_t$, and the foreign market offers a benchmark with return $v^F_t$. Both returns are independent across them and over time, and are normally distributed with mean $\bar{v}$ and variance $\sigma_v^2$. However, investors are only able to invest through mutual funds.

There are many managers, and each either invests in the domestic market, invests in the foreign market, or stays out. $N$ denotes the number of managers in the domestic market and $L$ the number of managers in the foreign market. The manager of the mutual fund has the ability to generate excess returns with respect to the benchmark. Let $R^M_{tj} = \alpha_j + v^M_t + \varepsilon_{tj}$ denote the return of manager $j$ in market $M \in \{D,F\}$, where $\alpha_j$ denotes the manager’s ability, and $\varepsilon_{tj}$ is the error term, which is normally distributed and independent over time and across managers, $\varepsilon_{tj} \sim N(0, \sigma^2)$ Each manager’s ability is normally distributed, $\alpha_j \sim N(\bar{\alpha}, \sigma^2)$, independent of other managers’ abilities, and is unknown to managers and investors. Managers are paid a fixed fee, $f$, per unit of capital they manage, and there is no cost of active management. Managers maximize the present discounted profit, which is equivalent to maximizing the present discounted value of received funds.

Ex ante, the only difference between the domestic and the foreign market is that the past performance of the domestic benchmark is observable, while the past performance of the foreign benchmark is not observable.

The first period is divided in three stages. First, managers decide simultaneously which market to enter, and there is a fixed cost of entry $F$, which is identical in both markets. Each manager can enter only one market and the entry decision is irreversible. Then, investors choose their optimal portfolio. Finally, returns are realized. In the second period, investors observe the realized returns of all mutual funds in both markets, $R^D_{t1} \equiv \left\{ R^D_{1j} \right\}_{j=1}^N$ and $R^F_{t1} \equiv \left\{ R^F_{1j} \right\}_{j=1}^L$, and the realization of the domestic benchmark, $v^D_t$. They update their belief about each manager’s ability and relocate their assets accordingly. There is no switching cost for investors.

The assumption that managers can operate only in one market is clearly a simplification, but we believe it is without much loss of generality. First, even if legally possible, it might be

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3If the cost of entry were higher in the foreign market, which is reasonable, our results would be strengthen. Given the higher cost of entry, foreign mutual funds must obtain more capital, which, as we show later, requires fewer firms in the foreign market.
disadvantageous for a manager to operate in many markets because there might be returns to scale in information processing. Second, even if the same manager operates in many markets, his or her ability to generate abnormal returns might be different for each market, and this is what is enough for our results to hold.\footnote{This assumption is supported by the empirical evidence provided in Hau and Rey (2008). They showed that the distribution of the markets in which mutual funds operate is bimodal. The distribution has a peak for completely home biased funds and a peak for funds operating only in foreign markets. This assumption could be theoretically rationalized with a model similar to the one proposed by Van Nieuwerburgh and Veldkamp (2008). Their model shows that there are increasing returns to scale to information processing when investors have a portfolio choice and an information processing choice.}

2.1 Portfolio choice

In this section, we study the portfolio choices of investors given the number of mutual funds in the domestic market, $N$, and the number of mutual funds in the foreign market, $L$. Let $q^D_{it}$ and $q^F_{it}$ denote the amount of capital invested by an individual $i$ at time $t$ in mutual funds in the domestic and the foreign markets respectively. The total amount of capital invested in the domestic market at time $t$ is given by $q^D_t = \int q^D_{it}di$, and the total amount of capital invested in the foreign market at time $t$ is given by $q^F_t = \int q^F_{it}di$. Since investors have one unit of capital, the investment satisfies $q^D_t + q^F_t = 1$. Let $x^D_{jt}$ be the investment received by manager $j$ in the domestic market at time $t$ and $x^F_{jt}$ be the investment received by manager $j$ in the foreign market at time $t$.

2.1.1 The first period

First, let us consider the portfolio choice in the first period. Since initially investors perceive all managers to be identical, all managers in a given market receive the same amount of funds:

$$x^D_{j1} = \frac{q^D}{N}, \quad x^F_{j1} = \frac{q^F}{L}. \quad (1)$$

Investors choose $q^D_1$ and $q^F_1$ to maximize their utility

$$\max_{q^D_1,q^F_1} q^D_1 (\bar{\nu} + \alpha - f) + q^F_1 (\bar{\nu} + \alpha - f) - \frac{1}{2} \gamma \left( (q^2_D + q^2_F) \sigma_\nu^2 + \left( \frac{q^2_D}{N} + \frac{q^2_F}{L} \right) \left( \sigma^2_\alpha + \sigma^2_\epsilon \right) \right),$$

\footnote{This assumption is supported by the empirical evidence provided in Hau and Rey (2008). They showed that the distribution of the markets in which mutual funds operate is bimodal. The distribution has a peak for completely home biased funds and a peak for funds operating only in foreign markets. This assumption could be theoretically rationalized with a model similar to the one proposed by Van Nieuwerburgh and Veldkamp (2008). Their model shows that there are increasing returns to scale to information processing when investors have a portfolio choice and an information processing choice.}
subject to the budget constraint \( q_{D1} + q_{F1} = 1 \). The optimal amount of funds invested in each market is given by

\[
q_{D1} = \frac{\sigma_\alpha^2 + \frac{1}{2} (\sigma_\alpha^2 + \sigma_\varepsilon^2)}{2 \sigma_\alpha^2 + (\frac{1}{2} + \frac{1}{2})(\sigma_\alpha^2 + \sigma_\varepsilon^2)}, \quad q_{F1} = \frac{\sigma_\alpha^2 + \frac{1}{2} (\sigma_\alpha^2 + \sigma_\varepsilon^2)}{2 \sigma_\alpha^2 + (\frac{1}{2} + \frac{1}{2})(\sigma_\alpha^2 + \sigma_\varepsilon^2)},
\]

(2)

The difference in the amount of capital invested in each market depends only on the number of mutual funds in that market. The market with a higher number of mutual funds is more diversified; and therefore, it receives more funds. The amount of capital received by one mutual fund is obtained by plugging equation (2) into equation (1). Equation (1) implies that each manager in the more diversified market receives less capital than each manager in the other market.

2.1.2 The second period

In the second period, investors update their beliefs about each manager’s ability. In the domestic market, investors observe the realized returns of all managers \( \{ R_{j1}^D \}_{j=1}^N \) and \( v_1^D \), and update

\[
\phi_{j1} (R_{j1}^D, v_1^D) \equiv E [ \alpha_j \mid R_{11}^F, \ldots, R_{1L}^F, v_1^D ] = E [ \alpha_j \mid R_{j1}^D, v_1^D ] = \frac{\tilde{\alpha} \sigma_\varepsilon^2 + \left( R_{j1}^D - v_1^D \right) \sigma_\alpha^2}{\sigma_\varepsilon^2 + \sigma_\alpha^2},
\]

(3)

\[
\sigma_{\alpha D}^2 \equiv Var (\alpha_i \mid R_{j1}^D, v_1^D) = \frac{\sigma_\alpha^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\alpha^2}.
\]

(4)

In the foreign market, investors observe the realized returns of all managers \( \{ R_{j1}^F \}_{j=1}^L \), but they do not observe \( v_1^F \); hence

\[
\phi_{j1} (R_{j1}^F) \equiv E [ \alpha_j \mid R_{11}^F, \ldots, R_{L1}^F ] = \tilde{\alpha} + \frac{[L \sigma_\varepsilon^2 + \sigma_\alpha^2 + \sigma_\varepsilon^2] \sigma_\alpha^2 (R_{j1}^F - \bar{\tilde{\alpha}}) - \sum_{k=1}^L \sigma_\varepsilon^2 \sigma_\alpha^2 (R_{k1}^F - \bar{\tilde{\alpha}})}{(\sigma_\varepsilon^2 + \sigma_\alpha^2) (L \sigma_\varepsilon^2 + \sigma_\alpha^2 + \sigma_\varepsilon^2)}
\],

(5)

\[
\sigma_{\alpha F}^2 \equiv Var (\alpha_j \mid R_{11}^F, \ldots, R_{L1}^F) = \frac{\sigma_\alpha^4 + \sigma_\varepsilon^4 (\sigma_\varepsilon^2 + \sigma_\alpha^2) + L \sigma_\varepsilon^2 \sigma_\alpha^2}{(\sigma_\alpha^2 + \sigma_\varepsilon^2)^2 (\sigma_\alpha^2 + \sigma_\varepsilon^2 + \sigma_\varepsilon^2)}
\],

(6)

where \( R_{1}^F \equiv \{ R_{j1}^F \}_{j=1}^L \).

In the domestic market investors observe the domestic benchmark, and the realizations of
the other managers’ returns do not carry additional information about particular manager’s ability. In the foreign market, on the other hand, since investors do not observe the foreign benchmark, the returns of all managers help predict the ability of a particular manager. As a result, investors get a much noisier information about each foreign manager’s ability; therefore, the uncertainty about the ability of the managers after the first period is higher in the foreign market, \( \sigma_{aF}^2 > \sigma_{aD}^2 \).

After updating their beliefs, investors choose the allocation of capital between the domestic and the foreign market, \( q_{D2} \) and \( q_{F2} \), and the allocation of funds within each market, \( \{x_{j2}^D\}_{j=1}^N \) and \( \{x_{j2}^F\}_{j=1}^L \). Given the amount of capital invested in each market, \( q_{D2} \) and \( q_{F2} \), investors choose \( \{x_{j2}^D\}_{j=1}^N \) as to solve

\[
\max_{\{x_{j2}^D\}_{j=1}^N} \sum_{j=1}^N \left( \bar{\nu} + \phi_{j1} \left( R_{j1}, v_1^D \right) - f \right) x_{j2}^D - \frac{1}{2} \gamma \left( \sum_{j=1}^N x_{j2}^D \right)^2 \sigma_v^2 + \sum_{j=1}^N \left( x_{j2}^D \right)^2 \left( \sigma_{aD}^2 + \sigma_{aF}^2 \right), \tag{7}
\]

subject to \( \sum_{j=1}^N x_{j2}^D = q_{D2} \). The allocation of funds in the domestic market is

\[
x_{j2}^D = \frac{q_{D2}}{N} + \frac{\phi_{j1} \left( R_{j1}, v_1^D \right) - \frac{1}{N} \sum_{i=1}^N \phi_{i1} \left( R_{i1}, v_1^D \right)}{\gamma \left( \sigma_{aD}^2 + \sigma_{aF}^2 \right)}. \tag{8}
\]

Analogously, the allocation of funds in the foreign market

\[
x_{j2}^F = \frac{q_{F2}}{L} + \frac{\phi_{j1} \left( R_{j1}, v_1^F \right) - \frac{1}{L} \sum_{i=1}^L \phi_{i1} \left( R_{i1}^F \right)}{\gamma \left( \sigma_{aF}^2 + \sigma_{aD}^2 \right)}. \tag{9}
\]

The optimal allocation of capital between the domestic and the foreign market is obtained by plugging the optimal distribution of capital across managers, which is given by equations (8) and (9), into the objective function given by equation (7). Then, investors choose \( q_{D2} \) and \( q_{F2} \) to maximize this objective function subject to the budget constraint \( q_{D2} + q_{F2} = 1 \). The optimal investment in each market is

\[
q_{D2} = \frac{\gamma \left( \sigma_v^2 + \frac{1}{L} \left( \sigma_{aF}^2 + \sigma_{aD}^2 \right) \right) + \frac{1}{N} \sum_{i=1}^N \phi_{i1} \left( R_{i1}, v_1^D \right) - \frac{1}{L} \sum_{i=1}^L \phi_{i1} \left( R_{i1}^F \right)}{\gamma \left( 2\sigma_v^2 + \frac{1}{L} \left( \sigma_{aF}^2 + \sigma_{aD}^2 \right) + \frac{1}{N} \left( \sigma_{aD}^2 + \sigma_{aF}^2 \right) \right)}. \tag{10}
\]
\[ q_{F2} = \frac{\gamma (\sigma_v^2 + \frac{1}{N} (\sigma_{D,D}^2 + \sigma_{D,E}^2)) - \frac{1}{N} \sum_{i=1}^{N} \phi_i (R_{D1}^P, v_1^D) + \frac{1}{N} \sum_{i=1}^{L} \phi_i (R_{F1}^P)}{\gamma (2\sigma_v^2 + \frac{1}{N} (\sigma_{F,D}^2 + \sigma_{F,E}^2) + \frac{1}{N} (\sigma_{D,D}^2 + \sigma_{D,E}^2))}. \] (11)

If the average expected ability in both markets after the first period is the same, investors channel more capital into the domestic market because the estimate of manager’s ability is more precise there. Each manager receives more capital when she is in an undiversified market, when the average quality in her market is relatively high compared with the other market, and when her quality is relatively high compared with the average quality in her market.

### 2.2 Market entry

In the initial stage, the managers decide simultaneously whether they enter the domestic market, the foreign market, or stay out. At this time, the managers know how much capital they will receive in the first period, but can only calculate the expected amount of funds they will receive in the second period. The latter is obtained by taking the unconditional expectations of equations (8) and (9). The ex-ante amount of capital that the managers receive in the second period if they enter the domestic market is

\[ E[x_{Dj}^2] = \frac{(\sigma_v^2 + \frac{1}{N} (\sigma_{D,D}^2 + \sigma_{D,E}^2))}{N (2\sigma_v^2 + \frac{1}{N} (\sigma_{F,D}^2 + \sigma_{F,E}^2) + \frac{1}{N} (\sigma_{D,D}^2 + \sigma_{D,E}^2))}, \] (12)

and if they enter the foreign market is

\[ E[x_{Fj}^2] = \frac{(\sigma_v^2 + \frac{1}{N} (\sigma_{D,D}^2 + \sigma_{D,E}^2))}{L (2\sigma_v^2 + \frac{1}{N} (\sigma_{F,D}^2 + \sigma_{F,E}^2) + \frac{1}{N} (\sigma_{D,D}^2 + \sigma_{D,E}^2))}. \] (13)

In equilibrium, it must be that no manager could attract more capital by switching from one market to the other; that is,

\[ x_{j1}^D (N, L) + \delta E [x_{j2}^D (N, L)] \geq x_{j1}^F (N - 1, L + 1) + \delta E [x_{j2}^F (N - 1, L + 1)], \]

\[ x_{j1}^D (N + 1, L - 1) + \delta E [x_{j2}^D (N + 1, L - 1)] \leq x_{j1}^F (N, L) + \delta E [x_{j2}^F (N, L)], \]

where \( \delta \) is a discounting factor, and the arguments denote the number of mutual funds in the domestic and the foreign market respectively.

In the appendix, we prove the following proposition.
Proposition 1  In equilibrium, there are more mutual funds in the domestic market, \( N \geq L \).

If there was the same number of mutual funds in each market, all mutual funds would attract the same amount of capital in the first period, but the domestic funds would expect to attract more in the second period. Hence, in equilibrium the competition in the domestic market must be stronger to counterbalance this effect.

The expected amount of total capital in the second period in the domestic market is

\[
E[q_{D2}] = \frac{\left(\sigma^2_v + \frac{1}{L}\left(\sigma^2_{aF} + \sigma^2_{z}\right)\right)}{\left(2\sigma^2_v + \frac{1}{L}\left(\sigma^2_{aF} + \sigma^2_{z}\right) + \frac{1}{N}\left(\sigma^2_{aD} + \sigma^2_{z}\right)\right)},
\]

and in the foreign market is

\[
E[q_{F2}] = \frac{\left(\sigma^2_v + \frac{1}{N}\left(\sigma^2_{aD} + \sigma^2_{z}\right)\right)}{\left(2\sigma^2_v + \frac{1}{L}\left(\sigma^2_{aF} + \sigma^2_{z}\right) + \frac{1}{N}\left(\sigma^2_{aD} + \sigma^2_{z}\right)\right)}.
\]

Proposition 2 states that in expectation the domestic market attracts more capital.

Proposition 2  There is home bias in both periods; i.e., \( q_{D1} > q_{F1} \) and \( E[q_{D2}] > E[q_{F2}] \).

Proof. Proposition 1 states that \( N \geq L \). This together with equation (2) implies that \( q_{D1} \geq q_{F1} \). In the second period, the expected amount of capital allocated in the domestic market, equation (14), is higher than the amount of capital allocated in the foreign market, equation (15), if and only if \( N \left(\sigma^2_{aF} + \sigma^2_{z}\right) \geq L \left(\sigma^2_{aD} + \sigma^2_{z}\right) \), which is always true since \( N \geq L \) and \( \sigma^2_{aF} > \sigma^2_{aD} \).

Home bias results from two effects: a direct one and an indirect one. First, investors have more precise information about managers’ ability in the domestic market; therefore, they can distribute their investments better than in the foreign market. This causes them to channel more capital to the domestic market even if the number of mutual funds in each market is the same. However, due to this primary home bias, more managers enter the domestic market, making it more diversified. This in turn causes the investors to invest even more in the domestic market; hence, home bias becomes even more severe.
3 Heterogeneous managers

In this section, we analyze a situation in which managers are not ex ante identical. The ability of each manager consists of a publicly observed signal, $y_j$ and an unknown, random factor $\eta_i$; that is, $\alpha_j = y_j + \eta_j$, where $y_j \sim N(\bar{\alpha}, \sigma_y^2)$ and $\eta_j \sim N(0, \sigma_\eta^2)$, and $y_j, \eta_j, y_i, \eta_i$ are independent for $i \neq j$. The signals are observed before managers choose in which market to operate. The rest of the game is the same as in the previous section. When we want to stress that we look at a manager in a particular market, we denote the signal of manager $j$ who enters market $M$ by $y_j^M$.

3.1 Allocation of capital by investors

Given the signal structure, in the first period the expected ability of manager $j$ is $y_j$. Let:

- $y^D = \frac{1}{N} \sum_{j=1}^{N} y_j^D$ be the average expected ability in the domestic market,
- $y^F = \frac{1}{L} \sum_{j=1}^{L} y_j^F$ be the average expected ability in the foreign market.

The first period optimal allocation of capital across markets is

- $q_{D1} = \frac{\gamma \left( \sigma_v^2 + \frac{1}{N} \left( \sigma_\eta^2 + \sigma_\varepsilon^2 \right) \right) + y^D - y^F}{\gamma \left( 2\sigma_v^2 + \left( \frac{1}{N} + \frac{1}{L} \right) \left( \sigma_\eta^2 + \sigma_\varepsilon^2 \right) \right)}$,

$$q_{F1} = \frac{\gamma \left( \sigma_v^2 + \frac{1}{L} \left( \sigma_\eta^2 + \sigma_\varepsilon^2 \right) \right) - y^D + y^F}{\gamma \left( 2\sigma_v^2 + \left( \frac{1}{N} + \frac{1}{L} \right) \left( \sigma_\eta^2 + \sigma_\varepsilon^2 \right) \right)};$$

and across managers is

- $x_{Dj} = \frac{\gamma \left( \sigma_v^2 + \frac{1}{N} \left( \sigma_\eta^2 + \sigma_\varepsilon^2 \right) \right) + y^D - y^F}{\gamma N \left( 2\sigma_v^2 + \left( \frac{1}{N} + \frac{1}{L} \right) \left( \sigma_\eta^2 + \sigma_\varepsilon^2 \right) \right)} + \frac{y_i - y^D}{\gamma \left( \sigma_\eta^2 + \sigma_\varepsilon^2 \right)}$,

$$x_{Fj} = \frac{\gamma \left( \sigma_v^2 + \frac{1}{L} \left( \sigma_\eta^2 + \sigma_\varepsilon^2 \right) \right) - y^D + y^F}{\gamma L \left( 2\sigma_v^2 + \left( \frac{1}{N} + \frac{1}{L} \right) \left( \sigma_\eta^2 + \sigma_\varepsilon^2 \right) \right)} + \frac{y_i - y^F}{\gamma \left( \sigma_\eta^2 + \sigma_\varepsilon^2 \right)};$$

The total amount of capital invested in a given market is increasing in the diversification of this market and in the difference between the average expected quality of managers in this market and the other market. Each manager is better off in a market that is not well diversified.
has higher average expected quality, and has lower expected average quality compared to the quality of that fund’s manager.

As before, in the second period investors update their beliefs about the ability of the managers. In the domestic market, investors observe signals about managers’ abilities, \( \{y_j\}_{j=1}^N \), the realized returns of all managers \( \{R_{j1}^D\}_{j=1}^N \) and the domestic benchmark, \( v_1^D \), and update their beliefs in the following way

\[
\phi_{j1} (y_j, R_{j1}^D, v_1^D) = E \left[ \alpha_j \mid \{y_j\}_{j=1}^N, R_{11}^D, ..., R_{N1}^D, v_1^D \right] = E \left[ \alpha_j \mid y_j, R_{j1}^D, v_1^D \right], \tag{20}
\]

\[
\sigma_{\alpha D}^2 \equiv Var \left( \alpha_i \mid y_j, R_{j1}^D, v_1^D \right) = \sigma_e^2 \frac{\sigma_n^2}{\sigma_e^2 + \sigma_n^2}.
\tag{21}
\]

In the foreign market, investors observe signals about managers’ abilities, \( \{y_j\}_{j=1}^L \), and the realized returns of all managers, \( \{R_{j1}^F\}_{j=1}^L \), but they do not observe the foreign benchmark, \( v_1^F \). They update the beliefs in the following way

\[
\phi_{j1} (\{y_j\}_{j=1}^L, R_1^F) = E \left[ \alpha_j \mid \{y_j\}_{j=1}^L, R_{11}^F, ..., R_{L1}^F \right] \tag{22}
\]

\[
\sigma_{\alpha F}^2 \equiv Var \left( \alpha_i \mid \{y_j\}_{j=1}^L, R_{11}^F, ..., R_{L1}^F \right) = \frac{\sigma_n^2}{\sigma_e^2 + \sigma_n^2} \frac{\sigma_e^4 + \sigma_n^2 (\sigma_e^2 + \sigma_n^2) + L \sigma_n^2 \sigma_e^2}{\sigma_e^2 + \sigma_n^2 + L \sigma_n^2}.
\tag{23}
\]

The second period optimal allocation of capital across markets is

\[
q_{D2} = \frac{\gamma (\sigma_o^2 + \frac{1}{L} (\sigma_{\alpha D}^2 + \sigma_e^2)) + \left( \frac{1}{N} \sum_{j=1}^N \phi_{j1} (y_j, R_{j1}^D, v_1^D) - \frac{1}{L} \sum_{j=1}^L \phi_{j1} (\{y_j\}_{j=1}^L, R_1^F) \right)}{\gamma (2 \sigma_o^2 + \frac{1}{L} (\sigma_{\alpha F}^2 + \sigma_e^2) + \frac{1}{N} (\sigma_{\alpha D}^2 + \sigma_e^2))},
\tag{24}
\]

\[
q_{F2} = \frac{\gamma (\sigma_o^2 + \frac{1}{N} (\sigma_{\alpha D}^2 + \sigma_e^2)) - \left( \frac{1}{N} \sum_{j=1}^N \phi_{j1} (y_j, R_{j1}^D, v_1^D) - \frac{1}{L} \sum_{j=1}^L \phi_{j1} (\{y_j\}_{j=1}^L, R_1^F) \right)}{\gamma (2 \sigma_o^2 + \frac{1}{L} (\sigma_{\alpha F}^2 + \sigma_e^2) + \frac{1}{N} (\sigma_{\alpha D}^2 + \sigma_e^2))},
\tag{25}
\]

and across managers is

\[
x_{j2}^D = \frac{1}{N} q_{D2} + \frac{\phi_{j1} (y_j, R_{j1}^D, v_1^D) - \frac{1}{N} \sum_{j=1}^N \phi_{j1} (y_j, R_{j1}^D, v_1^D)}{\gamma (\sigma_{\alpha D}^2 + \sigma_e^2)}, \tag{26}
\]
When deciding which market to enter, each manager should take into account her impact on the number of mutual funds in each market, $N$ and $L$, the transparency of the foreign market, $\sigma^2_{\alpha F}$, and the average expected ability in each market. This strategic interaction complicates the analysis significantly, especially because the equilibrium outcome depends crucially on a particular realization of signals. To be able to provide analytical insights into the problem, we focus on a particular type of the equilibria.

**Definition** A myopic entry equilibrium is an equilibrium in which at the location stage no manager has an incentive to change her decision if she believes that it will not affect $y^D, y^F, N$, and $L$.

One can easily see that, although a standard equilibrium in the entry stage always exists, there might not exist a myopic entry equilibrium. We discuss the implications of our equilibrium concept and the existence problem in section 3.3.1.

When deciding which market to enter, a manager with signal $y_j$ estimates how her ability will be perceived in the second period, and this estimate is the same in both markets:

$$E \left[ \phi_{j1} \left( y_j, R^D_{j1}, v^D_1 \right) \right] = E \left[ \phi_{j1} \left( \{y_j\}_{j=1}^L, R^F_1 \right) \right] = y_j.$$  

The ex-ante expected optimal allocation of capital received by a manager with $y_j$ in the second period is obtained using equations (26) and (27):

$$E \left[ x^D_{j2} \right] = \frac{\gamma \left( \sigma^2_\nu + \frac{1}{L} \left( \sigma^2_{\alpha F} + \sigma^2_\xi \right) \right) + (y^D - y^F)}{N \gamma \left( 2\sigma^2_\nu + \frac{1}{L} \left( \sigma^2_{\alpha F} + \sigma^2_\xi \right) + \frac{1}{N} \left( \sigma^2_{\alpha D} + \sigma^2_\xi \right) \right)} + \frac{(y_j - y^D)}{\gamma \left( \sigma^2_{\alpha D} + \sigma^2_\xi \right)};$$  

if she is in the domestic market, and

$$E \left[ x^F_{j2} \right] = \frac{\gamma \left( \sigma^2_\nu + \frac{1}{L} \left( \sigma^2_{\alpha D} + \sigma^2_\xi \right) \right) + (y^D - y^F)}{L \gamma \left( 2\sigma^2_\nu + \frac{1}{L} \left( \sigma^2_{\alpha F} + \sigma^2_\xi \right) + \frac{1}{N} \left( \sigma^2_{\alpha D} + \sigma^2_\xi \right) \right)} + \frac{(y_j - y^F)}{\gamma \left( \sigma^2_{\alpha F} + \sigma^2_\xi \right)};$$  

if she is in the foreign market.
In a myopic entry equilibrium, the net incentive to enter the domestic market instead of the foreign market for a manager with signal \( y_j \) is \( x_{Dj} - x_{Fj} \) in the first period and \( E \left[ x_{Dj}^2 \right] - E \left[ x_{Fj}^2 \right] \) in the second period. We have

\[
\frac{d \left( x_{Dj} - x_{Fj} \right)}{dy_j} = 0,
\]

\[
\frac{d \left( E \left[ x_{Dj}^2 \right] - E \left[ x_{Fj}^2 \right] \right)}{dy_j} = \frac{\sigma^2_{\alpha_F} - \sigma^2_{\alpha_D}}{\gamma \left( \sigma^2_{\alpha_D} + \sigma^2_{\epsilon} \right)} \left( \sigma^2_{\alpha_D} + \sigma^2_{\epsilon} \right) > 0.
\]

This means that the signals about ability do not affect the difference in the amount of capital a given fund receives in the first period. In the second period, however, the expected capital is increasing in the manager’s signal, but the impact of the signal is higher in the domestic market. The capital in the future depends on the second period evaluation of the manager’s ability, which in expectation is equal to \( y_j \). However, in the domestic market the evaluation of the manager’s ability will be more precise, hence will affect investors’ decisions more.

We can now state the following proposition.

**Proposition 3** If there exists a myopic entry equilibrium, then it is unique. In this equilibrium, for any realization of signals in the population of potential entrants there exist thresholds \( \bar{y}_1 \) and \( \bar{y}_2 \), with \( \bar{y}_1 > \bar{y}_2 \), such that all managers with \( y_j > \bar{y}_1 \) enter the domestic market and all managers with \( y_j \in (\bar{y}_2, \bar{y}_1) \) enter the foreign market.

**Proof.** First, if a manager with signal \( y_i \) enters one of the markets, then a manager with signal \( y_j > y_i \) will also benefit from entering that market. Now, assume that there exist two managers with signals \( y_i^D \) and \( y_i^F \), such that \( y_i^D < y_i^F \), and the first manager is in the domestic and the second in the foreign market. Then \( \delta E \left[ x_{j2}^D (y_i^D) - x_{j2}^F (y_i^D) \right] < \delta E \left[ x_{j2}^D (y_i^F) - x_{j2}^F (y_i^F) \right] \leq x_{j1}^F - x_{j1}^D \), where the last inequality follows from the fact that \( y_i^F \) prefers to be in the foreign market. Hence, the manager with signal \( y_i^D \) would also prefer to be in the foreign market. ■

The intuition for this is simple. In a domestic market investors estimate the ability of the managers with greater precision; hence, they react more to this estimate. Therefore, other things equal, managers with higher expected ability benefit more from being in the transparent market.
3.3 Home bias

We evaluate expected home bias after the entry stage. We say that there is home bias in period $t$ if $q_{Dt} > q_{Ft}$.

**Proposition 4** There is home bias in the second period.

**Proof.** Taking the expectations of equations (24) and (25), we obtain that $E[q_{D2}] > E[q_{F2}]$ if

$$2(y^D - y^F) > \gamma \frac{1}{N} (\sigma_{\alpha D}^2 + \sigma_{\xi}^2) - \gamma \frac{1}{L} (\sigma_{\alpha F}^2 + \sigma_{\xi}^2).$$

Clearly, if $N \geq L$, the right hand side of the above equation is negative; and hence, there is home bias. Assume then that $N < L$.

In equilibrium, for all managers in the domestic market

$$x_{j1}^D(y_j) + \delta E[x_{j2}^D(y_j)] > x_{j1}^F(y_j) + \delta E[x_{j2}^F(y_j)],$$

and for all managers in the foreign market

$$x_{j1}^D(y_j) + \delta E[x_{j2}^D(y_j)] < x_{j1}^F(y_j) + \delta E[x_{j2}^F(y_j)].$$

We have shown before that $x_{j1}^F(y_j) - x_{j1}^D(y_j)$ is independent of $y_j$; hence, in equilibrium we have

$$\delta E[x_{j2}^D(y_j) - x_{j2}^F(y_j)] < x_{j1}^F - x_{j1}^D < \delta E[x_{j2}^D(y_j) - x_{j2}^F(y_j)].$$

In particular, this has to be satisfied for $y_j^F = y_F$.

We have two cases: $E[x_{j2}^D(y_F) - x_{j2}^F(y_F)] < 0$ and $E[x_{j2}^D(y_F) - x_{j2}^F(y_F)] > 0$.

Case 1: Using (28) and (29), $E[x_{j2}^D(y_F) - x_{j2}^F(y_F)] < 0$ implies

$$2(y^D - y^F) > 2(\sigma_{\xi}^2 + \sigma_{\alpha D}^2) \gamma \frac{(L - N)\sigma_{\xi}^2 + (\sigma_{\alpha F}^2 - \sigma_{\alpha D}^2)}{N(\sigma_{\alpha F}^2 - \sigma_{\alpha D}^2 + 2L\sigma_{\xi}^2)}.$$

It is immediate to show that

$$2(\sigma_{\xi}^2 + \sigma_{\alpha D}^2) \gamma \frac{(L - N)\sigma_{\xi}^2 + (\sigma_{\alpha F}^2 - \sigma_{\alpha D}^2)}{N(\sigma_{\alpha F}^2 - \sigma_{\alpha D}^2 + 2L\sigma_{\xi}^2)} > \gamma \frac{1}{N}(\sigma_{\alpha D}^2 + \sigma_{\xi}^2) - \gamma \frac{1}{L}(\sigma_{\alpha F}^2 + \sigma_{\xi}^2);$$
hence, there is home bias in the second period.

Case 2: \( E \left[ x_{j1}^D \left( y^F \right) - x_{j2}^F \left( y^F \right) \right] > 0 \), which implies that \( x_{j1}^F - x_{j1}^D > 0 \). The latter implies that

\[
2 \left( y^D - y^F \right) > (\sigma_y^2 + \sigma_n^2) \gamma \left( \frac{1}{N} - \frac{1}{L} \right).
\]

Hence, we have a home bias if we can show that

\[
(\sigma_y^2 + \sigma_n^2) \left( \frac{1}{N} - \frac{1}{L} \right) > \frac{1}{N} (\sigma_{\alpha D}^2 + \sigma_{\epsilon}^2) - \gamma \frac{1}{L} (\sigma_{\alpha F}^2 + \sigma_{\epsilon}^2),
\]

but this is true because \( L > N \), and \( L (\sigma_n^2 - \sigma_{\alpha D}) > N (\sigma_n^2 - \sigma_{\alpha F}) \).

In the first period, there is home bias when \( x_{j1}^D < x_{j1}^F \), and this happens when there are more mutual funds in the domestic market. The equilibrium number of managers in each market, however, depends on a particular realization of the signals; hence we cannot draw general conclusions about the bias in the first period.

### 3.3.1 Robustness

In this section, we discuss the limitations of focusing on a myopic entry equilibrium. Clearly, as the number of managers entering each market grows, the difference between a standard equilibrium and our notion of the equilibrium becomes negligible. However, since for our findings to hold we need the number of managers to be finite, we lose some generality.

First, as we have mentioned above, a myopic entry equilibrium might not always exist. A manager in one market might find it profitable to deviate to the other market, but after doing this and observing new \( N \) and \( L \), she might want to reverse her decision. This should happen, however, only in the knife-edge cases.

We are more concerned with whether our results about the average ability in each market and home bias still hold when a standard notion of the equilibrium is used. When the managers ignore their impact on the toughness of the market, the difference between the benefit from entering the domestic and the benefit from entering the foreign market still depends on the conditions in both markets. However, the impact of the ability of the manager depends only on the transparency of each market: the higher the transparency, the more reward each manager gets for her ability. This implies that the only equilibrium is a threshold equilibrium, in which the managers with the highest signals go to the domestic market.
When each manager additionally takes into account that her entry affects the number of mutual funds in the market, there might be multiple equilibria, and in particular it might happen that some managers with higher signals enter the foreign market. This is simply because if there are enough managers in the foreign market, even a good ability manager might be better off in the foreign market or staying out. However, in a non-generic case the average expected ability in the domestic market will still be higher.

When each manager takes into account that her decision affects also $y_D$ and $y_F$, there is an additional effect. A manager always benefits if her ability is high relative to the average ability in her market. Her ability, however, affects the average ability more in the market with fewer mutual funds; hence, other things equal, a high ability manager prefers to enter the market with more mutual funds while low ability manager prefers a market with fewer mutual funds. If in equilibrium there are more mutual funds in the domestic market, this effect strengthens our results. If not, this effect works in the opposite direction to the previous ones, and it is not obvious that the additional benefit from entering the domestic market is still increasing in $y_j$. Given that the equilibrium number of mutual funds in each market depends on a particular realization of signals, we cannot say in general which effect will be stronger. However, using the intuition developed in the previous sections, we conjecture that the average ability in the domestic market should still be higher and there should be home bias.

4 Quantifying and Simulating the Model

In the previous section, we showed the existence of home bias in a myopic entry equilibrium. In this section, we run simulations to evaluate the implications of the model under a more standard equilibrium notion. In a standard market entry equilibrium, managers take into account their impact on the number of mutual funds in each market, $N$ and $L$, the transparency of the foreign market, $\sigma_{\alpha_F}^2$, and the average expected ability in each market.

The parameters in the simulation are chosen to match those calibrated in Berk and Green (2004). The expected manager’s ability to provide excess returns, $\bar{\alpha}$, is 6.5%, while the standard deviation of the manager’s ability, $\sigma_\alpha$, is 6 percent. As in Berk and Green (2004), we also set at 20% the standard deviation of the tracking error around the benchmark, $\sigma_\varepsilon$. We assume there are 10 managers that are willing to operate in either market. The standard
deviation of the benchmark return, \( \sigma_y \), is set at 20%. The standard deviation of the publicly observed signal about the ability of the manager, \( \sigma_y \), is 3%. The standard deviation of the tracking error of each manager’s ability is then found by taking into account \( \sigma_y^2 = \sigma_y^2 + \sigma_y^2 \).

Risk aversion is set at \( \gamma = 2 \) and the discount factor at \( \delta = 0.97 \).

We focus on equilibria in which higher quality managers are in the domestic market and the lower quality managers are in the foreign market. For each simulation, we draw 10 signals, order them, and place \( N \) highest signals in the domestic market and \( L = 10 - N \) in the foreign market. We search for \( N \) such that no manager has an incentive to deviate.\(^6\)

We run 10,000 simulations and we always find a unique \( N \) satisfying the equilibrium condition. In all these simulations, we find that the number of managers in each market is the same, \( N = L = 5 \). In both periods, we obtain that on average 73% of the capital is invested in domestic assets.\(^7\) Figure 1 presents a histogram with a summary of the results. We see that at least 57% of the initial capital is invested domestically, while the maximum amount of capital invested domestically generated by the model is 96%. The range of the percentage of capital invested in domestic markets predicted by the model in most simulations is between 68% and 77%.

\[ \text{Home Bias at the Fund Level} \]

\[ \begin{array}{c}
0 & 500 & 1000 & 1500 & 2000 & 2500 & 3000 & 3500 & 4000 \\
0.5 & 0.55 & 0.6 & 0.65 & 0.7 & 0.75 & 0.8 & 0.85 & 0.9 & 0.95 & 1
\end{array} \]

\[ \text{Frequency} \]

\[ \% \text{ of Domestic Assets in the First Period, } q_{D1} \]

\[ \% \text{ of Domestic Assets in the Second Period, } q_{D2} \]

Figure 1: This figure shows histograms for the percentage of capital invested in domestic assets in the first period and the second period. There are 10,000 simulations. In each simulation, we draw individual public signals, \( y_i \), for 10 managers. The parameters used in the simulation are the following: \( \sigma_\alpha = 0.06, \sigma_\varepsilon = 0.2 \),

\(^6\)The details of the algorithm used are in the appendix.

\(^7\)It is common practice in theoretical models to report the expected home bias generated by the model as a measure of success.
$\sigma_y = 0.03, \sigma_v = 0.2, \bar{\alpha} = 0.065, \gamma = 2$ and $\delta = 0.97$.

These results generated by the model are on average twelve percentage points smaller than the recent empirical results provided by Chan, Covrig and Ng (2005) and Hau and Rey (2008). They find that the aggregate mutual fund home bias in the US is 85%. Even though the model is not able to explain the whole puzzle by itself, it provides an explanation of why a large fraction of the capital is invested domestically at the mutual fund level.

5 Conclusion

Majority of personal investment is executed with the help of portfolio managers. Despite the fact that portfolio managers should have similar access to information about any financial market, there is a bias in individual investment in favor of domestic markets. We have shown that even if mutual funds have no advantage in domestic markets, the uncertainty about their ability to deliver returns might generate home bias. If individual investors have less information about the foreign markets, it is more difficult for them to evaluate the portfolio manager’s ability to deliver excess returns; and hence, the allocation of investor’s funds is less efficient. That causes them to scale back their foreign investment. When portfolio managers are ex ante identical, more of them will operate in the domestic market to take advantage of more funds being directed there. This additionally causes the foreign markets to be underdiversified, which makes the home bias more severe. If managers are ex ante heterogenous, managers with higher expected ability to generate excess returns will enter the domestic market, because a more transparent market rewards ability more. This also adds to home bias.

In addition to providing a plausible explanation for home bias in individual investment, our model predicts that mutual funds in the domestic market on average generate higher excess returns. It would be interesting to see whether the data confirms this finding.

6 Appendix

Proof of Proposition 1. Let $z^2_{\alpha F}$ be the conditional variance of $\alpha_j$ in the foreign market in the second period when there are $L - 1$ managers in this market. We have $z^2_{\alpha F} > \sigma^2_{\alpha F}$.
Using formulas 12, 13 and 1, we have

\[ x_{j1}^D (N + 1, L - 1) - x_{j1}^F (N, L) = \]

\[ = \sigma_v^2 (L - N - 1) \left( \frac{(\sigma_\alpha^2 + \sigma_\varepsilon^2) (L + N - 2) + 2N \sigma_\varepsilon^2 (L - 1)}{2\sigma_v^2 (N + 1) (L - 1) + (\sigma_\alpha^2 + \sigma_\varepsilon^2) (L + N))} \right) \]

and

\[ E [x_{j2}^D (N + 1, L - 1)] - E [x_{j2}^F (N, L)] = \]

\[ = \text{sign} \left( L - N - 1 \right) \left( 2N (L - 1) \sigma_v^4 + \left( N (z_{\alpha F}^2 - \sigma_{\alpha F}^2) + (L - 2) D + FN \right) \sigma_v^2 \right) + \]

\[ + \left( NL (z_{\alpha F}^2 - \sigma_{\alpha F}^2) + N (2L - 1) (\sigma_{\alpha F}^2 - \sigma_{\alpha D}^2) \right) \sigma_v^2 + \]

\[ + (z_{\alpha F}^2 - \sigma_{\alpha F}^2) ((L - 1) D + (\sigma_{\alpha F}^2 + \sigma_{\varepsilon}^2) (\sigma_{\alpha F}^2 - \sigma_{\alpha D}^2)) + \]

\[ + (\sigma_{\alpha D}^2 - \sigma_{\alpha F}^2) ((L - 1) (\sigma_{\alpha D}^2 + \sigma_{\varepsilon}^2) + (\sigma_{\alpha F}^2 + \sigma_{\varepsilon}^2) N) \],

where \( \text{sign} \) denotes that the expression which follows has the same sign as the expression that precedes.

Hence, the benefit from deviating from the foreign to the domestic market is

\[ x_{j1}^D (N + 1, L - 1) - x_{j1}^F (N, L) + \delta \left( E [x_{j2}^D (N + 1, L - 1)] - E [x_{j2}^F (N, L)] \right) = \]

\[ = A (L - N - 1) + (L - N - 1) B + C, \]

where \( A, B \) and \( C \) are some positive constants. In equilibrium, this benefit must be negative; hence, it must be that \( N \geq L \). ■

**Algorithm for the simulations**

We show the algorithm used for the simulations. We start by drawing 10 signals \( y_i \) from a normal distribution with mean 6.5% and the standard deviation of 3%. We order the managers from the best to the worst, and name them \( y_1, y_2, ..., y_{10} \). We put the \( N \) managers with the highest \( y_i \) in the domestic market and \( L = 10 - N \) in the foreign market.

Let \( s_{\alpha F}^2 \) be the variance in the foreign market when there are \( L + 1 \) managers and let \( z_{\alpha F}^2 \) be the variance in the foreign market if there are \( L - 1 \) managers there. Next, we define the payoffs of the managers in each market and the payoffs of the managers if they deviate.

1. For any manager \( i \) in the domestic market:
a. First Period: If managers stay in the domestic market, they obtain the following capital allocation

\[
x_{j1}^D = \frac{\gamma \left( \sigma_v^2 + \frac{1}{L} \left( \sigma_{\eta}^2 + \sigma_{\varepsilon}^2 \right) \right) + \left( \frac{\sum_{i \neq j} y_i^D + y_j}{N} - \frac{\sum_{i=1}^{L} y_i^F}{L} \right)}{\gamma N \left( 2\sigma_v^2 + \left( \frac{1}{L} + \frac{1}{N} \right) \left( \sigma_{\eta}^2 + \sigma_{\varepsilon}^2 \right) \right)} + \frac{\left( y_j - \frac{\sum_{i \neq j} y_i^D + y_j}{N} \right)}{\gamma \left( \sigma_{\eta}^2 + \sigma_{\varepsilon}^2 \right)}.
\]

If instead they deviate, they get

\[
d_{j1}^F = \frac{\gamma \left( \sigma_v^2 + \frac{1}{(N-1)} \left( \sigma_{\eta}^2 + \sigma_{\varepsilon}^2 \right) \right) - \left( \frac{\sum_{i \neq j} y_i^D + y_j}{N-1} - \frac{\sum_{i=1}^{L} y_i^F + y_j}{L+1} \right)}{\gamma (L+1) \left( 2\sigma_v^2 + \left( \frac{1}{L+1} + \frac{1}{N-1} \right) \left( \sigma_{\eta}^2 + \sigma_{\varepsilon}^2 \right) \right)} + \frac{\left( y_j - \frac{\sum_{i \neq j} y_i^D + y_j}{L+1} \right)}{\gamma \left( \sigma_{\eta}^2 + \sigma_{\varepsilon}^2 \right)}.
\]

The difference between both payoffs is increasing or decreasing in the manager’s ability depending on the sign of the following expression

\[
\frac{\partial \left( x_{j1}^D - d_{j1}^F \right)}{\partial y_i} = \left( \frac{\gamma N \left( 2\sigma_v^2 + \left( \frac{1}{L} + \frac{1}{N} \right) \left( \sigma_{\eta}^2 + \sigma_{\varepsilon}^2 \right) \right)}{\gamma} + \frac{\sum_{i \neq j} y_i^D + y_j}{N} \right) - \frac{\sum_{i=1}^{L} y_i^F}{L+1} - \frac{\left( \sigma_{\eta}^2 + \sigma_{\varepsilon}^2 \right)}{\gamma L+1}.
\]

b. Second Period: If managers stay in the domestic market, they obtain the following expected capital allocation

\[
E_1 \left[ x_{j2}^D | y_j \right] = \frac{\gamma \left( \sigma_v^2 + \frac{1}{L} \left( \sigma_{\alpha_F}^2 + \sigma_{\varepsilon}^2 \right) \right) + \left( \frac{\sum_{i \neq j} y_i^D + y_j}{N} - \frac{\sum_{i=1}^{L} y_i^F}{L} \right)}{N \gamma \left( 2\sigma_v^2 + \left( \frac{1}{L} \left( \sigma_{\alpha_F}^2 + \sigma_{\varepsilon}^2 \right) \right) + \frac{1}{N} \left( \sigma_{\alpha_D}^2 + \sigma_{\varepsilon}^2 \right) \right)} + \frac{\left( y_j - \frac{\sum_{i \neq j} y_i^D + y_j}{N} \right)}{\gamma \left( \sigma_{\alpha_D}^2 + \sigma_{\varepsilon}^2 \right)}.
\]

If instead they deviate, they get

\[
E_1 \left[ d_{j2}^F | y_j \right] = \frac{\gamma \left( \sigma_v^2 + \frac{1}{(N-1)} \left( \sigma_{\alpha_D}^2 + \sigma_{\varepsilon}^2 \right) \right) - \left( \frac{\sum_{i \neq j} y_i^D + y_j}{N-1} - \frac{\sum_{i=1}^{L} y_i^F + y_j}{L+1} \right)}{(L+1) \gamma \left( 2\sigma_v^2 + \left( \frac{1}{L+1} \left( \sigma_{\alpha_F}^2 + \sigma_{\varepsilon}^2 \right) \right) + \frac{1}{N-1} \left( \sigma_{\alpha_D}^2 + \sigma_{\varepsilon}^2 \right) \right)} + \frac{\left( y_j - \frac{\sum_{i \neq j} y_i^D + y_j}{L+1} \right)}{\gamma \left( \sigma_{\alpha_F}^2 + \sigma_{\varepsilon}^2 \right)}.
\]

The difference between both expected payoffs is increasing or decreasing in the manager’s
ability depending on the sign of the following expression

\[
d \left( E_1 \left[ x_{j1}^F | y_j \right] - E_1 \left[ d_{j1}^D | y_j \right] \right) = \left( N \gamma \left( 2 \sigma_0^2 + \frac{1}{L} \left( \sigma_{\alpha F}^2 + \sigma_\eta^2 \right) + \frac{1}{N} \left( \sigma_{\alpha D}^2 + \sigma_\xi^2 \right) \right) + \frac{N-1}{N} \gamma \left( \sigma_{\alpha D}^2 + \sigma_\xi^2 \right) \right) + \frac{1}{L+1} (L+1) \gamma \left( 2 \sigma_0^2 + \frac{1}{L+1} \left( s_{\alpha F}^2 + \sigma_\eta^2 \right) + \frac{1}{N-1} \left( \sigma_{\alpha D}^2 + \sigma_\xi^2 \right) \right) - \frac{L}{L+1} \gamma \left( s_{\alpha F}^2 + \sigma_\eta^2 \right).
\]

\[dy_i = dE_1 \left[ x_{j2}^F | y_j \right] = \frac{1}{L} \gamma L (2 \sigma_0^2 + \left( \frac{1}{L} + \frac{1}{N} \right) (\sigma_\eta^2 + \sigma_\xi^2)) + \frac{y_j - \frac{1}{N} \sum_{i=1}^{N-1} y_{L+i}^F + y_i}{\gamma (\sigma_\eta^2 + \sigma_\xi^2)}.
\]

If instead they deviate, they get

\[d_{j1}^D = \frac{\gamma \left( \sigma_0^2 + \frac{1}{L-1} \left( \sigma_\eta^2 + \sigma_\xi^2 \right) \right) + \frac{1}{N+1} \sum_{i=1}^{N-1} y_{L+i}^D + y_i}{\gamma (N+1) \left( 2 \sigma_0^2 + \left( \frac{1}{L-1} + \frac{1}{N+1} \right) (\sigma_\eta^2 + \sigma_\xi^2) \right)} + \frac{y_j - \frac{1}{N} \sum_{i=1}^{N} y_{L+i}^D + y_i}{\gamma (\sigma_\eta^2 + \sigma_\xi^2)}.
\]

The difference between both expected payoffs is increasing or decreasing in the manager’s ability depending on the sign of the following expression

\[d \left( x_{j1}^F - d_{j1}^D \right) = \left( \frac{1}{L} \gamma L (2 \sigma_0^2 + \left( \frac{1}{L} + \frac{1}{N} \right) (\sigma_\eta^2 + \sigma_\xi^2)) + \frac{L-1}{L} \gamma (\sigma_\eta^2 + \sigma_\xi^2) \right) + \frac{1}{N+1} \gamma (N+1) \left( 2 \sigma_0^2 + \left( \frac{1}{L-1} + \frac{1}{N+1} \right) (\sigma_\eta^2 + \sigma_\xi^2) \right) - \frac{N}{N+1} \gamma (\sigma_\eta^2 + \sigma_\xi^2).
\]

b. Second Period: If managers stay in the foreign market, they obtain the following
To this end, we look at the sign of the following expressions.

\[
E_1 \left[ x_{j2}^F | y_j \right] = \frac{\gamma \left( \sigma_0^2 + \frac{1}{N} \left( \sigma_{0D}^2 + \sigma_2^2 \right) \right) - \left( \sum_{i=1}^{N-1} y_i^F - \frac{\sum_{i=1}^{L-1} y_i^F + y_j}{L} \right)}{L \gamma \left( 2 \sigma_v^2 + \frac{1}{L} \left( \sigma_{vF}^2 + \sigma_v^2 \right) + \frac{1}{N} \left( \sigma_{vD}^2 + \sigma_v^2 \right) \right)} + \frac{y_j - \frac{\sum_{i=1}^{L-1} y_i^F + y_j}{L}}{\gamma \left( \sigma_{vF}^2 + \sigma_v^2 \right)}.
\]

If instead they deviate, they get

\[
E_1 \left[ d_{j2}^D | y_j \right] = \frac{\gamma \left( \sigma_0^2 + \frac{1}{L-1} \left( z_{aF}^2 + \sigma_2^2 \right) \right) + \left( \sum_{i=1}^{N} y_i^D + y_j \right) - \frac{\sum_{i=1}^{L-1} y_i^F + y_j}{L-1}}{(N+1) \gamma \left( 2 \sigma_v^2 + \frac{1}{L-1} \left( z_{aF}^2 + \sigma_v^2 \right) + \frac{1}{N+1} \left( \sigma_{aD}^2 + \sigma_v^2 \right) \right)} + \frac{y_j - \frac{\sum_{i=1}^{N} y_i^D + y_j}{N+1}}{\gamma \left( \sigma_{aF}^2 + \sigma_v^2 \right)}.
\]

The difference between both expected payoffs is increasing or decreasing in the manager’s ability depending on the sign of the following expression

\[
\frac{d \left( E_1 \left[ x_{j2}^F | y_j \right] - E_1 \left[ d_{j2}^D | y_j \right] \right)}{dy_i} = \left( \frac{1}{L} \frac{1}{\gamma \left( 2 \sigma_v^2 + \frac{1}{L} \left( \sigma_{vF}^2 + \sigma_v^2 \right) + \frac{1}{N} \left( \sigma_{vD}^2 + \sigma_v^2 \right) \right)} + \frac{L-1}{L} \right) - \frac{1}{(N+1) \gamma \left( 2 \sigma_v^2 + \frac{1}{L-1} \left( z_{aF}^2 + \sigma_v^2 \right) + \frac{1}{N+1} \left( \sigma_{aD}^2 + \sigma_v^2 \right) \right)} - \frac{N}{\gamma \left( \sigma_{aF}^2 + \sigma_v^2 \right)}.
\]

c. Managers in the foreign market deviate if

\[
d_{j1}^D + \delta E_1 \left[ d_{j2}^D | y_j \right] > x_{j1}^F + \delta E_1 \left[ x_{j2}^F | y_j \right]
\]

(31)

3. Next step is to find for each market the manager with the highest incentive to deviate. To this end, we look at the sign of the following expressions.

a. Domestic Market

\[
\frac{d \left( x_{j1}^D - d_{j1}^F \right)}{dy_i} + \delta \frac{d \left( E_1 \left[ x_{j2}^F | y_j \right] - E_1 \left[ d_{j2}^F | y_j \right] \right)}{dy_i}
\]

(32)

b. Foreign Market

\[
\frac{d \left( x_{j1}^D - d_{j1}^F \right)}{dy_i} + \delta \frac{d \left( E_1 \left[ x_{j2}^F | y_j \right] - E_1 \left[ d_{j2}^D | y_j \right] \right)}{dy_i}
\]

(33)

c. Find manager in each market with more incentives to deviate by using the following
For a given $N$, if for the two managers with the highest incentive to deviate conditions (30) and (31) are not satisfied, then $(N, 10 - L)$ constitute an equilibrium. If not, decrease the number of managers in the domestic market to $N - 1$ if a domestic manager wants to deviate and increase the number of managers in the domestic market to $N + 1$, if the foreign manager wants to deviate.

References


