When and How Should Infant Industries Be Protected?

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Abstract

This paper develops and analyzes a welfare maximizing model of infant industry protection. The domestic infant industry is competitive and experiences dynamic learning effects that are external to firms. The competitive foreign industry is mature and produces a good that is an imperfect substitute for the domestic good. A government planner can protect the infant industry by using domestic production subsidies, tariffs, and quotas in order to maximize domestic welfare over time. As protection is not always optimal (even though the domestic industry experiences a learning externality), the paper shows how the decision to protect the industry should depend on the industry’s learning potential, the shape of the learning curve, and the degree of substitutability between domestic and foreign goods.

Assuming some reasonable restrictions on the flexibility over time of the policy instruments, the paper subsequently compares the effectiveness of the different instruments. The economics literature has mainly explained the widespread use of quantity restrictions by appealing to non-welfare-maximizing behavior of governments or strategic interactions between firms and/or governments. In this work, quantity restrictions have been mostly interpreted as more distorting than other trade instruments such as tariffs or domestic production subsidies. This paper demonstrates that, under the model’s assumptions, the quota almost always yields higher welfare than the tariff. In some cases, the dominance of the quota is so pronounced that it compensates for any amount of government revenue loss caused by practical considerations involved in the administration of the quota. (This is true even in the extreme case of a voluntary export restraint, where no revenues would be collected.) It is further shown that the quota may even be preferred to a domestic production subsidy. The paper thus introduces a new argument to explain why quantity restrictions may create less distortions than tariffs or subsidies and can potentially be welfare enhancing.

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1 Introduction

The infant industry argument is one of the oldest arguments used to justify the protection of industries from international trade. First formulated by Alexander Hamilton and Friedrich List at the beginning of the Nineteenth Century, the case for infant industry protection has been generally accepted by economists over the last two centuries. Although some of the arguments supporting protection have come under successful attacks over the years, most economists would nonetheless agree to a list of specific circumstances that would warrant the temporary and limited protection of an infant industry. In his famous statement supporting the case for infant industry protection, John Stuart Mill alluded to one of the main circumstances on this list: the presence of dynamic learning effects that are external to firms.\(^1\) Mill recognized that certain additional conditions must also be met in order to justify protection. He specifically mentioned that protection must be temporary and that the infant industry must then mature and become viable without protection. Subsequently, Charles Francis Bastable added another condition requiring that the cumulative net benefits provided by the protected industry exceed the cumulative costs of protection.\(^2\) Together, these conditions are known as the Mill-Bastable Test. Economists currently recognize, with varying degrees of enthusiasm or skepticism, that the presence of these dynamic learning effects and the fulfillment of the Mill-Bastable Test constitute a valid case for infant industry protection. The economics literature has further shown that the protection provided by production subsidies is preferable to that provided by tariffs or quotas, as the latter additionally distort consumption. Nevertheless, production subsidies may not be feasible due to government fiscal constraints.

Now consider the problems encountered by a government planner who wishes to follow these relatively straightforward recommendations when deciding on a specific policy for an infant industry characterized by the previously mentioned learning effects. Though clear and intuitive, the Mill-Bastable Test is hard to apply in practice: both the benefits and costs of protection change over time as learning progresses. The cumulative benefits and costs not only reflect the changes driven by the learning process but also those caused by the adjustment over time of the level of protection (typically, the latter decreases as learning progresses). Recommendations for the policy instrument choice (subsidy, tariff, or quota) are equally clear but also greatly complicated by practical considerations. The recommendations are based on the assumption that the level of the

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\(^1\)See Mill (1848, pp. 918-19). The full statement is re-printed in Kemp (1960).

\(^2\)See Bastable (1891, pp. 140-143). For further discussion of the Mill-Bastable Test, see Kemp (1960) and Corden (1997, ch. 8).
policy instrument can be costlessly changed over time. In fact, these changes are costly and may not even be feasible over certain time intervals.\textsuperscript{3} How do these considerations affect the government planner’s choice of policy instrument?

This paper seeks to answer this question and assist the government planner with the application of the Mill-Bastable Test. The paper shows how the cumulative costs of protection can be approximated by a fixed learning cost that can be readily compared to an appropriately normalized benefit flow. The paper describes how the fulfillment of the test depends on the industry’s learning potential, the speed of learning, and the degree of substitutability between the domestic and foreign goods. When the test has been met, the paper then shows how the presence of adjustment costs and uncertainty concerning the learning curve confer an advantage to the quota over the other two policy instruments. In particular, the quota will almost always yield higher welfare outcomes than the tariff. In some cases, the dominance of the quota is so pronounced that it compensates for any amount of government revenue loss caused by practical considerations involved in the administration of the quota. (This is true even in the extreme case of a voluntary export restraint, where no revenues would be collected.) It is further shown that the quota may even be preferred to domestic production subsidies. Briefly, the advantage of the quota vis-a-vis the subsidy or tariff is that its level of protection automatically declines as learning progresses (a desired property for welfare maximization). On the other hand, the tariff and subsidy must be adjusted downward to produce this effect. This adjustment requires additional information about the pace of learning (which may not be known with certainty) and may be costly or even infeasible.

The economics literature has mainly explained the widespread use of quantity restrictions by appealing to models based on non-welfare-maximizing behavior by the government or based on strategic interactions among firms or between firms and the government.\textsuperscript{4} In this work, the use of quantity restrictions has been mostly interpreted as more distorting than other trade instruments such as tariffs or domestic production subsidies. The paper introduces a new argument into the trade policy literature that explains why quantity restrictions may create less distortions than tariffs or subsidies and can potentially be welfare enhancing. This argument supports a different interpretation of some of the past and current uses of quantity restrictions for infant industry protection and may inform recommendations for the design of future trade policies.

\textsuperscript{3}The cost or incapacity to adjust the policy instrument may be driven by actual costs and political procedures or alternatively by the capture of the political process (once the policy is implemented) by special interest groups.

\textsuperscript{4}These models are reviewed in Deardorff (1987). Political economy models that explain the use of trade policies as a voting or lobbying equilibrium also fall within this category.
2 Learning-by-Doing and Infant Industry Protection

I assume that the infant industry’s dynamic learning occurs through learning-by-doing. The infant industry argument based on this type of learning externality was first explicitly modeled in a dynamic framework by Bardhan (1971). His single industry model has since been extended to analyze the consequences of learning in more than one industry. Clemhout and Wan (1970) study infant industry protection policies for a group of industries that experience different rates of learning. Succar (1987) examines the impact of learning spillovers across industries. Krugman (1987) further extends the multi-industry model by allowing for learning in both the home and foreign industries. This last treatment departs from the assumption that a particular country is less developed than its trading partners and rather focuses on the study of the pattern of trade when comparative advantage is dynamic. His model does relate to infant industry protection, as he describes how a country can expand the set of industries in which it has a static comparative advantage through the use of trade policies. Redding (1999) incorporates welfare analysis in this type of model and explicitly shows how protection can enhance welfare through such a mechanism.

This paper returns to Bardhan’s single industry framework but relaxes the assumption that the domestic and foreign goods are perfect substitutes. Consumers thus derive some benefit from consuming both the domestic and foreign variety. This paper further extends his work by considering several different types of trade instruments when these are not perfectly flexible over time (Bardhan (1971) only considers the use of completely flexible production subsidies). Finally, this paper explicitly considers cases where the Mill-Bastable Test is not passed and examines how the characteristics of the industry influence the fulfillment of this test.

3 The Model

Learning and Production

The industry is comprised of perfectly competitive domestic firms who produce a homogeneous good that is an imperfect substitute for a foreign good whose supply is perfectly elastic at its constant marginal cost $\tilde{c}$ (throughout this paper, tildes (˜) will be used to represent foreign variables). The domestic industry’s technology exhibits static constant returns to scale, though the constant marginal cost $c_t$ decreases with cumulative industry production $Q_t$ as the industry is learning-by-
Let \( q_t \) and \( \bar{q}_t \) denote, respectively, the total domestic and foreign industry output at any time \( t \). I assume time to be continuous, so \( q_t = \dot{Q}_t \) and \( c_t = c(Q_t) \) where \( c(.) \) is the learning function. Learning ceases after a certain cumulative production \( \bar{Q} \) is attained at a positive marginal cost \( \bar{c} = c(\bar{Q}) \). The learning function is characterized by:

\[
\begin{align*}
  c(Q_o) &= c_o > \bar{c} \quad (Q_o = 0) \\
  c'(Q_t) &< 0 \quad \forall Q_t < \bar{Q} \\
  c(Q_t) &= \bar{c} \quad \forall Q_t \geq \bar{Q}
\end{align*}
\]

This learning function is assumed to be differentiable everywhere though its shape is not further restricted. The marginal cost lower bound \( \bar{c} \) can either be greater or lower than the constant foreign marginal cost \( \tilde{c} \) (The foreign industry has matured and does not experience any learning effects). The learning externality arises from the competitive nature of the industry: the firms assume that the effect of their own production on industry output is negligible and thus do not internalize the future cost-reducing effects of their current production. Each firm thus myopically values its output at its current marginal cost \( c_t \). Given a domestic subsidy \( \sigma_t \) and a tariff \( \tau_t \) at any time \( t \), the domestic and foreign supply to the domestic country will be perfectly elastic at domestic prices:

\[
    p_t = c_t - \sigma_t \quad \text{and} \quad \bar{p}_t = \tilde{c} + \tau_t.
\]

Since learning is only modeled in its reduced form, as incorporated in cumulative production, I only consider trade policies that directly affect the production level of the firms. As pointed out by Baldwin (1969), trade policies directed at the source of the externality (knowledge creation and dissemination) should be considered. This would involve a more structural approach to the learning process and is beyond the scope of this paper.

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5This type of learning further assumes that the spillover between firms is complete. See Dasgupta and Stiglitz (1988) for a discussion of the effects of incomplete vs. complete spillover.

6Throughout this paper, these outputs will represent the equilibrium quantities in the domestic market only. It is assumed that the domestic industry does not export while the static constant returns to scale and absence of foreign learning render the supply of the foreign good to other markets irrelevant to the domestic market equilibrium.

7Technically, \( c(.) \) can have a kink when learning ceases at \( \bar{Q} \) without changing any of the results.

8This type of extreme externality is assumed for computational ease. The qualitative results would remain unchanged if a learning externality were assumed where the firms do not fully internalize the benefits of current production on future costs.

9Given the competitive nature of the industry, a quota will have the same effect as an appropriately chosen tariff.
Domestic Demand

Domestic demand is generated by a representative consumer whose utility function is additively separable in a numeraire good.\(^\text{10}\) The utility gained from consumption of the domestic and foreign good is represented by a strictly concave function \(U(q_t, \tilde{q}_t)\). For simplicity, the consumer’s preferences over both goods are assumed to be symmetric - that is \(U(q_t, \tilde{q}_t) \equiv U(\tilde{q}_t, q_t)\). This will allow the cost side and trade policies to determine differences in consumption between the two goods. Let \(U_q(q_t, \tilde{q}_t)\) and \(U_{\tilde{q}}(q_t, \tilde{q}_t)\) denote the first partial derivatives and let \(U_{qq}(q_t, \tilde{q}_t), U_{\tilde{q}\tilde{q}}(q_t, \tilde{q}_t), U_{q\tilde{q}}(q_t, \tilde{q}_t)\) denote the second partial derivatives of the utility function \(U(q_t, \tilde{q}_t)\).\(^\text{11}\)

In order to make the analysis of the Mill-Bastable Test interesting, domestic consumers should be able to do without the foreign good and consume solely the domestic variety. The marginal benefit provided by consumption of the foreign good must therefore be bounded. I thus assume the existence of a finite price \(P^o\) such that \(P^o = U_q(0, 0) = U_{\tilde{q}}(0, 0)\). Regardless of the price of one of the goods, there will be no demand for the other if its price is above \(P^o\) (\(P^o\) is assumed to be high enough that \(P^o > \bar{c}\) and \(P^o > \tilde{c}\)). This assumption of a finite \(P^o\) is only important for the analysis of the Mill-Bastable Test. The remaining analysis describing interior solutions where both goods are consumed is not affected by this assumption.

The assumption of bounded marginal utility will imply the existence of “choke” prices (the price ceiling above which demand is zero). Since the two goods are related, the choke price for one good will depend on the price of the other as well as the degree of substitutability between the two goods. Let \(p^o(p)\) represent the choke price for either good as a function of the price of the other good. \(p^o(p)\) must be non-decreasing as the two goods are substitutes. The previous assumption on \(P^o\) further implies that \(p^o(p)\) must be constant at \(P^o\) for any price \(p\) above \(P^o\).\(^\text{12}\) When the two goods are unrelated, the choke price for one good will not depend on the price of the other: \(p^o(p)\) will be constant at \(P^o\) for all \(p\). Finally, \(p^o(p)\) falls as the goods become closer substitutes for any fixed \(p < P^o\). The graph of \(p^o(p)\) is shown in Figure 1 for different degrees of substitutability between the two goods. For simplicity, the graph of \(p^o(p)\) is drawn assuming that \(U(., .)\) is quadratic and hence that \(p^o(p)\) is linear.\(^\text{13}\)

\(^{10}\) The demand side of this model is very similar to Cheng (1988) who also studied optimal trade policies, though in the context of a static oligopoly model.

\(^{11}\) \(U_{q\tilde{q}}(q_t, \tilde{q}_t) = U_{\tilde{q}q}(q_t, \tilde{q}_t)\) by symmetry, while \(U_{q\tilde{q}}(q_t, \tilde{q}_t)^2 \leq U_{qq}(q_t, \tilde{q}_t)^2\) by concavity. \(U_{q\tilde{q}}(q_t, \tilde{q}_t) \leq 0\) since the goods are substitutes.

\(^{12}\) If the price of one good is above \(P^o\), there must be positive demand for the other at prices below \(P^o\) (since the utility gained from consuming the first unit is \(P^o\)).

\(^{13}\) The degree of substitutability captures the same concept as the elasticity of substitution between the two goods.
Given positive prices $p_t$ and $\tilde{p}_t$, and assuming that a positive quantity of the numeraire good is consumed, the demand for the two goods is derived from the consumer’s maximization of consumer surplus

$$\max_{q_t, \tilde{q}_t} CS_t = U(q_t, \tilde{q}_t) - p_t q_t - \tilde{p}_t \tilde{q}_t$$

Let $q(p_t, \tilde{p}_t)$ and $\tilde{q}(p_t, \tilde{p}_t)$ be the solution to this maximization problem. Further define $q^o(p_t) = q(p_t, P^o)$ as the demand function for the domestic good in autarky. By symmetry, this will also be the demand function for the foreign good when the domestic good is not available.\(^{14}\)

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\(^{14}\)Formally, $q^o(p_t)$ is the quantity $q^o_t$ such that $U_q(q^o_t, 0) = p_t$ and $U_{\tilde{q}}(0, q^o_t) = \tilde{p}_t$ for $p_t < P^o$. $q^o(p)$ is 0 when $p_t \geq P^o$. 

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Figure 1: Choke Price Function $p^o(p)$ for Different Degrees of Substitutability
Figure 2: Price Regions for Domestic Demand

\[ q(p_t, \tilde{p}_t) \text{ and } \tilde{q}(p_t, \tilde{p}_t) \text{ will then be given by:} \]

**case (I):**
\[
\begin{align*}
q(p_t, \tilde{p}_t) &= 0 \\
\tilde{q}(p_t, \tilde{p}_t) &= q^0(\tilde{p}_t)
\end{align*}
\]
if \( p_t \geq p^0(\tilde{p}_t) \),

**case (II):**
\[
\begin{align*}
q(p_t, \tilde{p}_t) &= q^0(p_t) \\
\tilde{q}(p_t, \tilde{p}_t) &= 0
\end{align*}
\]
if \( \tilde{p}_t \geq p^0(p_t) \),

**case (III):**
\[
\begin{align*}
q(p_t, \tilde{p}_t) &= q_t \\
\tilde{q}(p_t, \tilde{p}_t) &= \tilde{q}_t
\end{align*}
\]
subject to
\[
\begin{align*}
U_q(q_t, \tilde{q}_t) &= p_t \\
U_{\tilde{q}}(q_t, \tilde{q}_t) &= \tilde{p}_t
\end{align*}
\]
if \( p_t < p^0(\tilde{p}_t) \), \( \tilde{p}_t < p^0(p_t) \).

Note that the first two cases may simultaneously be satisfied, which implies zero demand for both goods. The space of possible price pairs \((p_t, \tilde{p}_t) \in \mathbb{R}_+^2\) can thus be partitioned into 4 regions as shown in Figure 2.\(^{15}\) When \( p_t < p^0(\tilde{p}_t) \) and \( \tilde{p}_t < p^0(p_t) \), there will be positive demand for both goods. When only one of these inequalities is reversed, there will be positive demand for only one good. Finally, when both inequalities are reversed, which can occur only when both prices are above \( P^a \), there will be no demand for either good.

\(^{15}\) Again, for simplicity, the regions are drawn assuming that \( U(\cdot, \cdot) \) is quadratic.
Domestic Welfare

Now suppose that a government social planner maximizes the sum of discounted domestic welfare over a given period of time $T$. Available to the government planner are trade policy instruments (a tariff, $\tau$, or domestic subsidy, $\sigma$) whose usage may potentially be restricted. At any point in time, a quota will be equivalent to a certain tariff level (I assume that the rights to a quota are auctioned by the planner and that the proceeds are collected by the government). At any time $t$, domestic welfare is given by the sum of consumer surplus and tariff revenue, minus the subsidy payment:

$$W_t = CS_t - \sigma_t q_t + \tau_t \tilde{q}_t = U(q_t, \tilde{q}_t) - c_t q_t - \tilde{c} \tilde{q}_t.$$ 

As pointed out by Dixit (Supplement 1984), the planner’s choice of trade instruments can be re-interpreted as a choice of consumption quantities from a feasible set generated by the consumer’s demand. Any restrictions on trade instruments would then be transformed into additional restrictions on this set of feasible consumption pairs. The planner’s problem can thus be written as:

$$\max_{(q_t, \tilde{q}_t) \in \mathcal{F}_t} \int_0^T e^{-\rho t} W_t \, dt = \int_0^T e^{-\rho t} \left( U(q_t, \tilde{q}_t) - c_t q_t - \tilde{c} \tilde{q}_t \right) \, dt,$$

where $\mathcal{F}_t \subseteq \mathbb{R}^2_+$ is the set of feasible quantity pairs at time $t$ and $\rho$ is the exogenous discount rate. The period of time $T$ is assumed to be long enough to cover the entire learning phase of the domestic industry given domestic production at the optimal level $q^*_t$ (as defined in (1)) as long as $q^*_t$ is positive.\textsuperscript{16}

4 No Intervention Benchmark

When no trade policy instruments are used, there will be three general cases characterizing the time paths of the consumption quantities. If the initial domestic cost $c_o$ is too high ($c_o \geq p^o(\bar{c})$), then the domestic industry will never produce, no matter what its learning potential may be. Consumption of the foreign good will then be constant at $\tilde{q}_t = q^o(\bar{c})$. When $c_o < p^o(\bar{c})$, consumption of the domestic good will be positive and increasing as learning drives down domestic costs to $\bar{c}$. As consumption of the domestic good rises, consumption of the foreign good drops. If $\bar{c}$ is low enough ($\bar{c} \geq p^o(\bar{c})$), then consumption of the foreign good will drop to zero at the time $t$ when $p^o(c_t) = \bar{c}$. Otherwise, if $\bar{c}$ is

\textsuperscript{16}The social planner also cares about domestic welfare after time $T$ though the optimal trade policy from that point on must be free trade.
“close enough” to $\tilde{c},^1^7$ then the consumption of the foreign good will drop but remain positive. After learning ceases, consumption levels of both goods will be constant at $\tilde{q} = q(\tilde{c}, \tilde{\tilde{c}})$ and $\tilde{\tilde{q}} = q(\tilde{c}, \tilde{\tilde{c}})$.

5 Fully Flexible Trade Policies

In this section, I assume that the government planner can choose any time paths for the subsidies and tariffs over the planning horizon from $t = 0$ to $t = T$. This assumption defines the set $\mathcal{F}_t$ of possible output pairs $(q_t, \tilde{q}_t)$. The social planner thus solves the maximization problem in (1) subject to the conditions $c_t = c(Q_t), q_t = \dot{Q}_t$ and the initial boundary condition $Q_0 = 0$ ($Q_T$ is left unrestricted). Let $\mathcal{H}$ be the current value Hamiltonian associated with this problem:

$$\mathcal{H}(q_t, \tilde{q}_t, Q_t) = U(q_t, \tilde{q}_t) - c(Q_t)q_t - \tilde{c}\tilde{q}_t + \lambda_t q_t$$

where $\lambda_t \geq 0$ is the current value shadow price of a unit of cumulative production.

Interior Solution

I first focus on the description of any interior solution to this problem. In this case, the first order conditions are:

$$\frac{\partial \mathcal{H}}{\partial q_t} = U_q(q_t, \tilde{q}_t) - c(Q_t) + \lambda_t = 0,$$  \hspace{1cm} (2)

$$\frac{\partial \mathcal{H}}{\partial \tilde{q}_t} = U_{\tilde{q}}(q_t, \tilde{q}_t) - \tilde{c} = 0,$$  \hspace{1cm} (3)

$$\frac{\partial \mathcal{H}}{\partial Q_t} = \rho \lambda_t - \dot{\lambda}_t.$$  \hspace{1cm} (4)

Since $Q_T$ is left unrestricted, the transversality condition yields $\lambda_T = 0$. Given an interior solution where $p_t = U_q(q_t, \tilde{q}_t)$ and $\tilde{p}_t = U_{\tilde{q}}(q_t, \tilde{q}_t)$, the first two conditions can be re-written as

$$\sigma_t = \lambda_t \quad \text{and} \quad \tau_t = 0,$$

which implies that, along an optimal interior path, no tariffs should be used and the subsidy in any period should be equal to the current value of an extra unit of cumulative production.

Condition (4) gives an equation of motion for domestic output. Using conditions (2) and (3), $\tilde{q}_t$,

17“close enough” implies $\tilde{c} < p^o(\tilde{c})$ and $\tilde{\tilde{c}} < p^o(\tilde{c})$. These conditions depend on the levels of $\tilde{c}$ and $\tilde{\tilde{c}}$, but also on the degree of substitutability between the goods.
λ_t, and \( \dot{\lambda}_t \) can be written as functions of \( Q_t, q_t, \) and \( \dot{q}_t \), yielding a second order differential equation for \( Q_t \). More intuition can be gained from this condition by integrating it:

\[
\int_t^T e^{-\rho s} \left( \rho \lambda_s - \dot{\lambda}_s \right) ds = -\int_t^T e^{-\rho s} c'(Q_s) q_s ds,
\]

which yields

\[
\left[ e^{-\rho s} \lambda_s \right]_t^T = -\left[ e^{-\rho s} c(Q_s) \right]_t^T - \int_t^T \rho e^{-\rho s} c(Q_s) ds,
\]

or

\[
\lambda_t = c_t - e^{-\rho(T-t)} c_T - \rho \int_t^T e^{-\rho(s-t)} c_s ds. \tag{5}
\]

Given the initial assumption on the length of the planning period \( T \) and the positive levels of domestic production, learning will cease at some time \( \bar{t}^* \) during the planning period.\(^{18}\) Thus, \( c_t = \bar{c} \) for \( \bar{t}^* \leq t \leq T \). Given the bounds \( \bar{c} \leq c_s \leq c_t \) for any \( s \) between \( t \) and \( T \), equation (5) will give bounds on the optimal subsidy \( \sigma_t = \lambda_t \):

\[
e^{-\rho(T-t)} \left( c_t - \bar{c} \right) \leq \lambda_t \leq c_t - \bar{c}. \tag{6}
\]

Thus, as long as there is learning potential \( (c_t > \bar{c}) \), a positive subsidy will be optimal though the subsidy should never be greater than this learning potential as measured by the cost difference between current cost and lowest potential cost. Of course, subsidies should cease with the learning at \( \bar{t}^* \): the bounds on \( \lambda_t \) both collapse to and remain at zero for any \( t \) at or beyond \( \bar{t}^* \). Using equation (5), it can be shown that, regardless of the shape of the learning function, the optimal subsidy will decrease throughout the learning phase, as long as the discount rate \( \rho \) is low enough.\(^{19}\)

For any discount rate, the optimal price path will be decreasing throughout the learning phase,\(^{20}\) implying an increasing domestic output path: subsidies should be used to increase domestic production, but initial subsidies should never be so high as to cause domestic output to decrease at any point in the future. The bounds derived for the optimal subsidy in (6) can be used to find bounds on the optimal price path:\(^{21}\)

\[
\bar{c} \leq p_t \leq e^{-\rho(\bar{t}^*-t)} \bar{c} + (1 - e^{-\rho(\bar{t}^*-t)}) c_t. \tag{7}
\]

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\(^{18}\)Of course, \( \bar{t}^* \) is endogenously determined by the planner.

\(^{19}\)This upper bound on \( \rho \) will depend on the shape of the learning function \( c(.) \). A sufficient condition for the decreasing subsidy is that costs \( c_t \) fall at a decreasing rate along the optimal path \( (\bar{c}_t \geq 0) \).

\(^{20}\)Since \( \dot{\lambda}_t = \dot{c}_t = \bar{c} - \dot{p}_t \), (4) can be written as \( \dot{p}_t = -\rho \sigma_t \leq 0 \).

\(^{21}\)\( \bar{t}^* \) is substituted for \( T \) since \( c_t = \bar{c} \) for \( t \geq \bar{t}^* \).
The upper bound on \( p_t \) is a weighted average of the current cost \( c_t \) and the lower bound cost \( \bar{c} \). For any reasonable discount factor and learning period length, the weight on the lower bound cost \( \bar{c} \) will dominate the weight on the current cost \( c_t \).\(^{22}\) If the discount rate is low enough, then this upper bound will be only a little higher than \( \bar{c} \) and the optimal price path will be almost constant at \( \bar{c} \). When the discount rate is zero, the optimal price path will be exactly constant at \( \bar{c} \).

Finally, the optimal interior consumption paths are obtained by applying the demand functions \( q(p_t, \tilde{p}_t) \) and \( \tilde{q}(p_t, \tilde{p}_t) \) to the optimal price paths. Since the optimal \( \tilde{p}_t \) path is constant at \( \tilde{c} \) and the optimal \( p_t \) path is decreasing, consumption of the domestic good increases while consumption of the foreign good decreases throughout the learning period. When the discount rate is low, these consumption paths will be almost constant at their post-learning levels of \( \bar{q} = q(\bar{c}, \tilde{c}) \) and \( \tilde{\bar{q}} = \tilde{q}(\bar{c}, \tilde{c}) \).

Given the assumed flexibility of the trade instruments, the feasibility of these consumption paths is guaranteed as long as the optimal subsidies generate positive demand for both goods. That is, at any time \( t \), the optimal price \( p_t \) must satisfy \( p_t < p^o(\tilde{c}) \) and \( \tilde{c} < p^o(p_t) \). If the discount rate is negligible,\(^{23}\) then these conditions are equivalent to \( \bar{c} < p^o(\tilde{c}) \) and \( \tilde{c} < p^o(\bar{c}) \). The lowest potential cost \( \tilde{c} \) must be “close enough” to the foreign cost \( \tilde{c} \).\(^{24}\)

**Corner Solutions**

The previous analysis assumes that consumption levels of both goods are strictly positive. It is also possible for one (but not both) of these quantities to be zero.

**No Consumption of Foreign Good**

If \( \tilde{c} > p^o(\bar{c}) \), then the optimal subsidies will eliminate demand for the foreign good during all or part of the learning phase. Once demand for the foreign good is driven to zero, it must remain at zero for the rest of the planning period. The optimal path of the subsidies will still be determined by the equation of motion (4).\(^{25}\)

\(^{22}\) A reasonable discount factor is assumed to be below 4\% and a reasonable learning period is assumed to last less than 5 years. Given these restrictions, the weight on the lower bound cost \( \bar{c} \) would always be at least 4 times higher than the weight on the current cost \( c_t \). Note that the planning period could extend well past the learning phase \((T \gg \bar{t}^*)\).

\(^{23}\) A negligible discount rate is low enough such that the effects of discounting on the optimal paths and welfare flows during the learning period can be ignored.

\(^{24}\) Again, “close enough” depends on the degree of substitutability. This requirement becomes more stringent as the goods become closer substitutes. When they are perfect substitutes, then close enough implies \( \bar{c} = \tilde{c} \).

\(^{25}\) The only difference in the first order conditions will be that the marginal utility of the foreign good is no longer equated to its marginal cost \( \tilde{c} \).
**No Consumption of Domestic Good**

If consumption of the domestic good is zero at any point along the optimal path, then it must be zero over the entire optimal path.\(^{26}\) Given this, the first order conditions only require that the marginal utility of the foreign good be equated to its cost (again, no tariff should be used) and that the shadow value of cumulative production be zero. Throughout the planning period, no subsidies should ever be used, and consumption of the foreign good would remain constant at \(\tilde{q}_t = q^0(\tilde{c})\). These consumption paths will be feasible and satisfy the first order conditions as long as the initial domestic cost \(c_o\) is high enough to preclude demand for the domestic good with no subsidies \((c_o \geq p^0(\tilde{c}))\).\(^{27}\) On the other hand, if the domestic cost lower bound is too high \((\bar{c} \geq p^0(\tilde{c}))\), then this corner solution will be the only possible one.

**The Mill-Bastable Test**

The previous analysis described how protection, if used, should optimally be applied. The analysis also revealed the existence of corner solutions where no protection is applied. Under certain conditions \((c_o \geq p^0(\tilde{c})\) and \(\bar{c} < p^0(\tilde{c})\)), both solutions will be possible (they both satisfy the first order conditions): the social planner must apply the Mill-Bastable Test and determine which solution yields the higher cumulative welfare.

**Total Welfare Over the Learning Period**

The evaluation of different trade policies critically depends on the comparisons of the welfare flows \(W_t\) induced by the trade policies. Unfortunately, these flows are hard to compare because they are not only a function of the consumption levels \((q_t, \tilde{q}_t)\) but also critically depend on the current level of domestic cost \(c_t\). This section shows how total welfare \(TW\) can be decomposed into a sum of welfare flows that do not depend on the changing current domestic cost and a separate fixed learning cost. For simplicity, the following derivations assume that the effects of discounting are

\(^{26}\)When the domestic good is consumed, its consumption path must be non-decreasing. Due to the concavity of the utility function, the optimal consumption path must be smooth and can not jump from a positive level down to zero. Thus, if consumption of the domestic good is positive at any time \(t\), it must also be positive at any time after \(t\). Furthermore, it can not be optimal to start domestic production at time \(t > 0\) since the same consumption path pushed back to start at \(t = 0\) would yield higher welfare. Hence, the possibility of the domestic not being produced (at any time \(t\)) can only occur when it is never produced.

\(^{27}\)Otherwise, demand for the domestic good would always be positive even with no government intervention and the only optimal solution must be the interior one previously described.
negligible.\(^{28}\) Given this assumption, and given any pair of consumption paths \((q_t, \tilde{q}_t)\) such that learning occurs before the end of the planning period \((Q_T \geq \bar{Q})\), total welfare can be decomposed in the following way:

\[
TW = \int_0^T W_t \, dt \\
= \int_0^T U(q_t, \tilde{q}_t) - c(Q_t)q_t - \tilde{c}\tilde{q}_t \, dt \\
= \int_0^T U(q_t, \tilde{q}_t) - \tilde{c}q_t - \tilde{c}\tilde{q}_t \, dt - \int_0^T (c(Q_t) - \tilde{c})q_t \, dt \\
= \int_0^T U(q_t, \tilde{q}_t) - \tilde{c}q_t - \tilde{c}\tilde{q}_t \, dt - \int_0^{\bar{Q}} (c(Q) - \tilde{c}) \, dQ,
\]

where the last line is obtained by changing the integration variable from \(t\) to \(Q\). Define a new welfare flow function \(\bar{w}(q_t, \tilde{q}_t) = U(q_t, \tilde{q}_t) - \tilde{c}q_t - \tilde{c}\tilde{q}_t\) which is the welfare flow assuming that the current domestic cost is constant at \(\tilde{c}\) instead of \(c_t\). Further define the fixed learning cost \(FLC\) as \(\int_0^{\bar{Q}} (c(Q) - \tilde{c}) \, dQ\). This cost is fixed because it does not depend on the chosen consumption paths but only on the cost reduction potential \((c_0 - \tilde{c})\) and the shape of the learning function: it is the area below the learning curve above the lowest potential cost line at \(\tilde{c}\). Given a cumulative production learning range determined by \(\bar{Q}\) and cost bounds \(c_o\) and \(\tilde{c}\), \(FLC\) will be large if learning is slow and small if learning is fast as shown in Figure 3.

Total welfare \(TW\) can thus be written as:

\[
TW = \int_0^T \bar{w}(q_t, \tilde{q}_t) \, dt - FLC. \tag{8}
\]

Regardless of the chosen consumption paths \((q_t, \tilde{q}_t)\), the total welfare generated by these paths during the planning period can be evaluated by using the static welfare flows \(\bar{w}(q_t, \tilde{q}_t)\) and subtracting the same fixed cost \(FLC\). This re-formulation will allow a straightforward comparison of different trade policy scenarios: if the domestic industry does not produce, then the welfare flows are constant at \(W^o = U(0, q^o(\tilde{c})) - \tilde{c}q^o(\tilde{c}) = \bar{w}(0, q^o(\tilde{c}))\) and no fixed learning cost is incurred. If the domestic industry produces, then the same welfare function \(\bar{w}(q_t, \tilde{q}_t)\) is used to evaluate the

\(^{28}\)The formal modeling of a small discount rate does not qualitatively change any of the results. The appendix describes in more detail how discounting will affect the following analysis. Briefly, the discount rate should be small enough such that the effects of discounting during the learning period are dominated by the learning effects. The effects of discounting after learning has occurred may be significant if the planning period extends significantly past the learning phase. The appendix shows how these effects can be included in the analysis without affecting the results.
welfare flows and the fixed learning cost is subtracted. Given that learning occurs, the optimal trade policy can ignore the learning costs and only seek to maximize the accumulated welfare flows \( w(q_t, \tilde{q}_t) \). This helps to explain the optimal subsidy path previously derived under the assumption of a negligible discount rate: since \( w(q_t, \tilde{q}_t) \) is concave and attains its global maximum at \((\bar{q}, \tilde{q})\), the highest possible welfare flows will be generated by keeping \( q_t \) and \( \tilde{q}_t \) constant at their long-run levels \( \bar{q} \) and \( \tilde{q} \). These were precisely the optimal consumption paths induced by the optimal subsidies. In other words, when the discount rate is negligible, the marginal social cost of the domestic good at any time is constant at its long run level \( \bar{c} \). Socially efficient consumption thus requires valuation of the domestic good at this lower price both during and after the learning period.

Let \( \overline{W} = \overline{w}(\bar{q}, \tilde{q}) \) be the maximum value of the welfare flow function \( \overline{w}(\cdot, \cdot) \). The determination of the Mill-Bastable Test – that is, between subsidization and no protection (and no domestic industry) – thus depends on the weighing of the higher welfare flows \( \overline{W} \geq W^o \) (the benefits of learning) against the fixed learning cost \( FLC \).

**Comparative Statics**

As the length of the planning horizon is extended, the subsidization alternative clearly becomes more attractive, because the benefits of the post-learning higher welfare are enjoyed over a longer time period. More interestingly, a low potential cost \( \bar{c} \) does not automatically entail that subsidization is optimal: the low \( \bar{c} \) can be offset by a high initial cost \( c_o \) or a slow learning curve, which both
increase the subsidization cost $FLC$ without affecting the difference between the welfare flows $\overline{W}$ and $W^o$. There will always exist a level of initial cost $c_o$ high enough and slow enough learning such that no subsidization is optimal even with arbitrarily low potential cost $\tilde{c}$.

The degree of substitutability between the two goods also affects the relative merits of subsidization. In order to vary the degree of substitutability while ensuring positive demand for both goods under optimal subsidization, I assume that $\tilde{c} = \bar{c}$ (this is also a reasonable assumption that a common technology and factor price equalization will prevail in the long run and equalize costs). Then, as the products become closer substitutes, the difference between $\overline{W}$ and $W^o$ decreases and goes to zero as the goods become perfect substitutes. On the other hand, the learning cost $FLC$ does not change with the degree of substitutability. Assuming that subsidization is optimal if the goods are unrelated (the initial cost $c_o$ is not too high), then the relative advantage of subsidization will continually decrease as the degree of substitutability rises until the subsidization scenario is no longer optimal. Clearly, subsidization will never be optimal when the goods are close enough substitutes, as the learning cost $FLC$ will always outweigh the small welfare advantage $\overline{W} - W^o$. Interestingly, the degree of substitutability critically affects the fulfillment of the Mill-Bastable Test but has no effect on the optimal path of the subsidies, given that protection is optimal.

6 Second Best Intervention: Subsidization no longer possible

Given budgetary or political constraints, subsidies may become infeasible, leaving only tariffs as the instrument available to the social planner. The level of the tariff is still assumed (in this section) to be fully flexible over time. In the previous section, given the existence of a domestic industry, subsidies were used to increase the production levels of the domestic good. Tariffs obviously share some substitutability as an instrument with the now unavailable subsidies since they can also raise domestic production above the free trade levels. On the other hand, it is well known that tariffs also induce an extra distortion on the consumption side: they drive a wedge between the social marginal cost of the foreign good (which is equal to the private marginal cost $\tilde{c}$ since no learning occurs) and its consumption price. The social planner must now trade off the benefits of higher domestic production against this new distortion.

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29 This is still assuming equality of long run costs ($\bar{c} = \tilde{c}$). If $\tilde{c}$ is significantly below $\bar{c}$ then subsidization may be optimal even when the two goods are perfect substitutes. 
30 Some of these constraints could also potentially alter the welfare function. This possibility is not considered in this paper. 
31 As long as there is some degree of substitutability between the goods and imports are positive.
As previously discussed, the social planner’s choice of trade policy (in this case, reduced to the choice of tariffs) is mapped into the equivalent choice of consumption pairs \((q_t, \tilde{q}_t)\) in a now restricted feasible set \(F_t\). The social planner’s maximization problem remains as in (1) with a new feasible set \(F_t\). When subsidies were feasible, it was always possible to induce domestic production, if necessary, with an appropriately chosen subsidy. Without subsidies, tariffs can be used to induce domestic production (when none would occur under no protection) in only a restricted set of cases: if the initial domestic cost is above the absolute choke price \(P^o\), then no tariff can induce domestic production.

**Interior Solution**

Given positive consumption of the domestic good, the absence of subsidies will imply marginal cost pricing of the domestic good or \(U_q(q_t, \tilde{q}_t) = c(Q_t)\). The consumption path of the foreign good is thus completely determined by the consumption path of the domestic good. The current value Hamiltonian \(\mathcal{H}\) can be re-written as a function of only the domestic equilibrium quantity:

\[
\mathcal{H}(q_t, Q_t) = U(q_t, \tilde{q}_t) - c(Q_t)q_t - \tilde{c}q_t + \mu_t q_t,
\]

where \(\tilde{q}_t\) is implicitly defined as a function of \(q_t\) and \(Q_t\) by \(U_q(q_t, \tilde{q}_t) = c(Q_t)\) and \(\mu_t\) is the new shadow price of cumulative domestic production. The new first order conditions are:

\[
\frac{\partial \mathcal{H}}{\partial q_t} = U_q(q_t, \tilde{q}_t) - c(Q_t) + \mu_t + U_{\tilde{q}}(q_t, \tilde{q}_t) \frac{\partial \tilde{q}_t}{\partial q_t} - \tilde{c} \frac{\partial \tilde{q}_t}{\partial q_t} = 0,
\]

\[
\frac{\partial \mathcal{H}}{\partial Q_t} = -c'(Q_t)q_t + U_{\tilde{q}}(q_t, \tilde{q}_t) \frac{\partial \tilde{q}_t}{\partial Q_t} - \tilde{c} \frac{\partial \tilde{q}_t}{\partial Q_t} = \rho \mu_t - \dot{\mu}_t,
\]

which, using \(p_t = U_q(q_t, \tilde{q}_t) = c(Q_t)\) and \(\tilde{p}_t = U_{\tilde{q}}(q_t, \tilde{q}_t) = \tilde{c} + \tau_t\), can be re-written as

\[
\tau_t = \eta_t \mu_t,
\]

\[
-c'(Q_t)q_t + \gamma_t \mu_t = \rho \mu_t - \dot{\mu}_t,
\]

where \(\eta_t = \left(\frac{\partial \tilde{q}_t}{\partial q_t}\right)^{-1} = \frac{U_{\tilde{q}}(q_t, \tilde{q}_t)}{U_q(q_t, \tilde{q}_t)} \in [0, 1]\) and \(\gamma_t = -\frac{\partial \tilde{q}_t}{\partial Q_t} \left(\frac{\partial \tilde{q}_t}{\partial q_t}\right)^{-1} = \frac{c'(Q_t)}{c'(Q_t)} \geq 0\). The equation of motion (10) is quite similar to the one previously obtained when subsidies were possible, indicating that the value of an additional unit of cumulative production, \(\mu_t\), will similarly decrease as learning.
progresses (when the discount rate is low).\textsuperscript{32} It can also be integrated in a similar way, yielding

\[
\mu_t = c_t - e^{-\rho(t^*-t)} \bar{c} - \rho \int_t^{t^*} e^{-\rho(s-t)} c_s \, ds + \int_t^{t^*} e^{-\rho(s-t)} \gamma_s \mu_s \, ds,
\]

which, assuming a negligible discount rate, can be written as

\[
\mu_t = (c_t - \bar{c}) + \int_t^{t^*} \gamma_s \mu_s \, ds.
\]

With a negligible discount rate, the value of an extra unit of cumulative production is always higher than it was when subsidies were possible, given the same level of cumulative production $Q_t$. This difference is due to the extra distortional effects of the tariff: recall from (8) that total welfare can be written as the accumulated welfare flows $\overline{w}(q_t, \tilde{q}_t)$ minus the fixed learning cost $FLC$. With subsidies, the welfare flows $\overline{w}(q_t, \tilde{q}_t)$ were maximized at every point in time – there was no distortion of consumption. The value of an extra unit of $Q_t$, $\lambda_t$, was just the incremental change in the fixed learning cost, which was just $(c_t - \bar{c})$. Without subsidies, the optimal consumption bundle $(\bar{q}, \bar{\tilde{q}})$ is no longer feasible (throughout the learning period) and the welfare flows $\overline{w}(q_t, \tilde{q}_t)$ will be reduced from their maximum value at $\overline{W}$ – this is the consumption distortion. A lower domestic cost $c_t$ will permit a higher welfare flow $\overline{w}(q_t, \tilde{q}_t)$. An extra unit of $Q_t$ will thus yield the same incremental reduction in $FLC$, $(c_t - \bar{c})$, but will additionally yield an incremental reduction in the consumption distortion. The value of this incremental reduction will be equal to $\int_t^{t^*} \gamma_s \mu_s \, ds$. As expected, this distortion decreases as learning progresses and the domestic cost $c_t$ approaches $\bar{c}$.

The other first order condition (9) shows the relationship between the optimal tariff and the value of cumulative production $\mu_t$. As long as the demand system generated by $U(q_t, \tilde{q}_t)$ is not too convex, the optimal tariff must be decreasing with $\mu_t$ as learning progresses.\textsuperscript{33} Again, the tariff must reach zero when learning ceases. Equation (9) also shows how the optimal path of the tariff is critically affected by the level of product differentiation (recall that in the previous case, the optimal subsidies were unaffected by the level of product differentiation). $\eta_t$ decreases with the level of product differentiation starting at its upper bound of one when the two goods are perfect substitutes and reaching its lower bound of zero when the two goods are unrelated. In this case, optimal tariffs will be zero: even though an extra unit of cumulative production is valuable

\textsuperscript{32}Of course, $\mu_t$ must be zero after learning ceases.

\textsuperscript{33}This will be the case for any quadratic $U(\cdot, \cdot)$ or C.E.S. sub-utility function $(U(q_t, \tilde{q}_t) = V_1(V_2(q_t, \tilde{q}_t))$ where $V_1$ is concave and $V_2$ is C.E.S)
$(\mu_t > 0)$, a tariff will not be able to raise domestic production and can only distort consumption. Furthermore, since $\mu_t \geq (c_t - \bar{c})$ for low discount rates, $\eta_t(c_t - \bar{c})$ will be a lower bound for the tariff at any time $t$. As the goods become better substitutes, $\eta_t$ rises and increases this lower bound. Thus, tariffs should be used as long as the goods have some degree of substitutability.

The optimal consumption paths are now harder to characterize, as both the tariffs and domestic costs are decreasing. The effect of the cost decrease will be to raise consumption of the domestic good and lower that of the foreign good, while the tariff decrease will have the opposite effect on both quantities. If the magnitudes of the decreases in the tariff and in domestic costs are “close enough,” then both quantities will be rising. On the other hand, it will never be possible for both quantities to be simultaneously decreasing. Of course, consumption of the domestic good must also always be below its autarky level $q^o(c_t)$. Finally, if the demand systems generated by $U(q_t, \tilde{q}_t)$ are not too non-linear and the discount rate is low, it can be shown that the optimal consumption paths for both goods should always be at or below their respective long run levels $\bar{q}$ and $\tilde{\bar{q}}$: consumption of the foreign good above $\tilde{\bar{q}}$ would imply that the domestic industry is not being protected enough and that welfare can be improved by increasing protection. Consumption of the domestic good above $\bar{q}$ would imply that the domestic industry is being protected too much.

**Corner Solutions**

Again, the possibilities of zero consumption of either the domestic or foreign good must be evaluated.

**No Consumption of Foreign Good**

The optimal consumption level of the foreign good can be driven to zero for two reasons: a sufficiently low domestic cost $c_t$ (in which case tariffs no longer have any effect) or a sufficiently high optimal tariff $\tau_t$. In this case, any tariff above $p^o(c_t) - \bar{c}$ would induce zero demand for the foreign good and yield identical outcomes. In any case, as long as the demand for the domestic good is positive, all of the conditions derived above remain valid when there is no demand for the foreign good.

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34Once more, “close enough” will depend on the degree of substitutability.
No Consumption of Domestic Good

As was true when subsidies were feasible, the only possibility of no domestic production (at any time $t$) occurs when the domestic good is never produced. Again, the first order conditions for this solution require that the value of cumulative production and the tariff be zero everywhere. Consumption of the foreign good will then be constant at $q^o(\tilde{c})$. These consumption paths will be feasible and satisfy the first order conditions as long as the initial domestic cost is high enough to preclude domestic demand under no intervention ($c_o \geq p^o(\tilde{c})$). In these cases, the social planner refuses to protect the domestic industry. Once again, if $\tilde{c} \geq p^o(\tilde{c})$, this scenario will be the only solution satisfying the first order conditions. Also, as previously noted, if the initial domestic cost is above the absolute choke price, then no tariff level can induce consumption of the domestic good and the only possibility is to consume only the foreign good. Without subsidies, domestic production can not be induced, even in situations where large potential welfare gains are at stake.

The Mill-Bastable Test

Once again, there will be ranges for both the initial domestic cost and its lower bound where the no-intervention solution and the optimal tariff solution both satisfy the first order conditions. This will happen when $\tilde{c} < p^o(\tilde{c})$ and $p^o(\tilde{c}) \leq c_o < P^o$: the initial cost is high enough to preclude domestic demand without protection but not so high as too make protection ineffective. In these cases, the social planner must again apply the Mill-Bastable Test and compare cumulative welfare under the two scenarios.

Total Welfare Over the Learning Period

In this section, I again assume that the discount rate is negligible with the same disclaimers (see note 28). Thus, when the domestic industry is protected, the social planner chooses the optimal tariffs to maximize total welfare over the planning period:

$$\max_{(q_t, \tilde{q}_t) \in \mathcal{F}_t} TW = \max_{(q_t, \tilde{q}_t) \in \mathcal{F}_t} \int_0^T \overline{w}(q_t, \tilde{q}_t) \, dt - FLC$$

where $\mathcal{F}_t$ is the (restricted) set of consumption pairs $(q_t, \tilde{q}_t)$ such that $U_q(q_t, \tilde{q}_t) = c_t$. As previously noted, the unrestricted maximand of $\overline{w}(\cdot, \cdot)$, $(\tilde{q}, \tilde{q})$, will not be in $\mathcal{F}_t$ as long as $c_t$ is above $\tilde{c}$ (unless the domestic and foreign good are perfect substitutes): $\overline{w}(q_t, \tilde{q}_t)$ will always be below $\overline{W}$ until learning ceases. It can further be shown that $\overline{w}(q_t, \tilde{q}_t)$ will always be increasing along the optimal
path, reaching $\bar{W}$ at $\bar{t}^*$ when learning ceases. Nevertheless, since it would always be possible to attain $W^o$, the optimal consumption paths $(q^*_t, \tilde{q}^*_t)$ must satisfy:

$$T \cdot W^o \leq \int_0^T \overline{w}(q^*_t, \tilde{q}^*_t) \, dt \leq T \cdot \bar{W} \quad (11)$$

Comparative Statics

Inequality (11) indicates that the decision to protect the infant industry with tariffs involves the same tradeoffs as the previous choice when subsidies were feasible: higher welfare gains from protection must be weighed against the fixed learning cost. Of course, protection will never be as valuable since the welfare gains will be lower and the learning cost is unchanged. This means that there will be a range of situations where subsidy protection is optimal but tariff protection (even when possible) is not.

The previously derived comparative statics involving the domestic cost bounds and the learning curve will be similar since these affect the fixed learning cost in an identical way. Similarly, an increase in the length of the planning horizon $T$ will again favor the protection alternative. The degree of substitutability will be the only exogenous parameter that affects this tradeoff differently. When subsidies were possible, a decrease in the degree of substitutability always increased the relative merits of subsidization against no-intervention. Without subsidies, a similar decrease in substitutability will have two opposite effects: it will increase the potential welfare benefits of protection by increasing the difference between $\bar{W}$ and $W^o$, but it will also simultaneously reduce the effectiveness of tariff protection.

7 Fixed Tariffs and Quotas

In this section, I continue to assume that subsidization is infeasible. I further assume, due either to political constraints or to adjustment costs, that changing the level of a tariff or quota over time is costly. I initially assume that these costs are high enough that the social planner is constrained to pick only one tariff or quota level for the entire planning period.\footnote{This assumption on the large size of the adjustment cost is initially used for simplicity and will subsequently be relaxed. The qualitative results rely only on the presence of some non-negligible adjustment cost. The possibility of indexing the tariff to the current cost conditions is addressed in a later section.} Given the competitive industry structure, a quota will be equivalent to a tariff at any point in time, though the value of this equivalent tariff will be changing through time when the value of the quota is fixed. A fixed quota
and fixed tariff will thus clearly have different dynamic properties, though both cases are subsumed in the previous section dealing with flexible tariffs.

Given these added restrictions on the use of trade instruments, the social planner must choose a trade instrument (quota or tariff), as well as determine its optimal level. Since this level remains fixed, these two choices will completely determine the consumption paths throughout the entire planning period. The set of feasible consumption pairs $\mathcal{F}_t$ is thus reduced to a set of trajectories indexed by the choice of instrument and its level. Of course, the social planner can also choose not to protect the infant industry. In this case, the domestic industry would still produce (and learning would occur) if the initial cost was low enough ($c_o < p^0(\tilde{c})$). I initially assume that this is the case in order to eliminate the possibility of no domestic production and learning.

Given these trade policy flexibility restrictions and the previous analysis of the optimal flexible tariff, the intuition for the superiority of the quota over the tariff is straightforward. A natural choice of quota level would be the long run consumption level of the foreign good $\bar{q}$. This fixed quota will generate a path over time for the equivalent tariff path that decreases throughout the learning period and remains at zero right after learning ceases: this is a good first approximation to the optimal flexible tariff path. On the other hand, the problems with a fixed tariff are also clear: any fixed tariff will either not offer enough protection early in the learning phase, or will have to protect too much later at the end of the learning phase. In fact, any positive fixed tariff will necessarily protect too much after learning ceases when no protection is optimal. This tariff will then induce consumption distortions from that point on, until the end of the planning period. These motivations for the superiority of the quota remain valid when discounting is introduced and for any type of utility function $U(q_t, \tilde{q}_t)$. In the following section, I show how the fixed quota dominates a fixed tariff for the case of negligible discounting and quadratic utility.\(^{36}\)

**Tariff versus Quota**

Given that the domestic industry produces and learning occurs, the same fixed learning cost will be incurred and any two scenarios can be evaluated by comparing the accumulated welfare flows of $\bar{w}(q_t, \tilde{q}_t)$, or $\int_0^T \bar{w}(q_t, \tilde{q}_t) \, dt$, generated by each scenario. In order to do this, certain properties of $\bar{w}(q_t, \tilde{q}_t) = U(q_t, \tilde{q}_t) - \bar{c}q_t - \tilde{c}\tilde{q}_t$ will be needed.

\(^{36}\)The appendix addresses the modeling of more general functional forms for utility and non-negligible discounting.
Properties of \( \overline{w}(q_t, \tilde{q}_t) \)

Define \( \overline{w}(p_t, \tilde{p}_t) \) to be the welfare flow measured as a function of prices instead of quantities: 
\[
\overline{w}^*(p_t, \tilde{p}_t) = \overline{w}\left(q(p_t, \tilde{p}_t), \tilde{q}(p_t, \tilde{p}_t)\right).
\]
Since no subsidies are possible, the domestic price will be fixed at the current domestic cost \( c_t \). Given \( c_t \), an incremental change in the tariff affects this welfare flow in the following way:

\[
\frac{\partial \overline{w}^*(c_t, \tilde{c} + \tau)}{\partial \tau} = U_q \frac{\partial q}{\partial p} - \tilde{c} \frac{\partial \tilde{q}}{\partial p} + U_{\tilde{q}} \frac{\partial \tilde{q}}{\partial p} - \tilde{c} \frac{\partial \tilde{q}}{\partial p}
\]

\[
= (c_t - \tilde{c}) \frac{\partial \tilde{q}}{\partial p} + \tau \frac{\partial \tilde{q}}{\partial p}
\]

\[
= (c_t - \tilde{c}) \frac{\partial \tilde{q}}{\partial p} + \tau \frac{\partial \tilde{q}}{\partial p}.
\]

(12)

With quadratic utility, the demand function \( \tilde{q}(..) \) will also be linear in both its arguments and \( \frac{\partial \overline{w}^*(c_t, \tilde{c} + \tau)}{\partial \tau} \) will be a linear function of the tariff. Define \( \tau^*(c_t) \) as the tariff level that maximizes \( \overline{w}^*(c_t, \tilde{c} + \tau) \) given \( c_t \):

\[
\tau^*(c_t) = \frac{-\frac{\partial \tilde{q}}{\partial p}(c_t - \tilde{c})}{\frac{\partial \tilde{q}}{\partial p}(c_t - \tilde{c})} = \frac{U_{\tilde{q}} q}{U_q q}(c_t - \tilde{c}).
\]

\( \overline{w}^*(c_t, \tilde{c} + \tau) \) will initially increase as \( \tau \) increases from zero, reaching a maximum when \( \tau = \tau^*(c_t) \), and then will decrease as the tariff increases past \( \tau^*(c_t) \). Using the envelope theorem, it can further be shown that the maximized welfare flow \( \overline{w}^*(c_t, \tilde{c} + \tau^*(c_t)) \) increases as the domestic cost \( c_t \) decreases.\(^3\) Furthermore, since \( \tilde{q}(..) \) is linear in both its arguments, (12) can be written as:

\[
\frac{\partial \overline{w}^*(c_t, \tilde{c} + \tau)}{\partial \tau} = (c_t - \tilde{c}) \frac{\partial \tilde{q}}{\partial p} + \tau \frac{\partial \tilde{q}}{\partial p}
\]

\[
= \tilde{q}(c_t, \tilde{c} + \tau) - \tilde{q}(c, \tilde{c})
\]

\[
= \tilde{q}(c_t, \tilde{c} + \tau) - \tilde{q}.
\]

This derivative will be zero when consumption of the foreign good is equal to \( \tilde{q} \). Thus, given any level of domestic cost \( c_t \), the tariff that maximizes the welfare flow \( \overline{w}^*(c_t, \tilde{c} + \tau), \tau^*(c_t) \), is precisely the tariff that induces consumption of the foreign good at its long run level \( \tilde{q} \). The maximum value of this welfare flow when the foreign quantity remains fixed at \( \tilde{q} \) will then increase as \( c_t \) decreases.

\(^3\)When \( U(..) \) is quadratic, \( \overline{w}^*(c_t, \tilde{c} + \tau) \) will be a globally concave second order polynomial in \( \tau \), assuring that the first order condition identifies the unique maximand.

\(^3\)Since \( \frac{\partial \overline{w}^*(c_t, \tilde{c} + \tau^*(c_t))}{\partial c_t} = \frac{\tilde{q} - \tilde{c} \tau^*(c_t)}{\tilde{q} q} \leq 0 \)
Welfare Comparisons

The intuitive advantages of the fixed quota at \( \bar{q} \) were previously discussed. I now show that this fixed quota must yield higher total welfare than any small fixed tariff (including the case of free trade). I then consider the case of an arbitrarily large fixed tariff and show that it will also always be dominated by an appropriately chosen fixed quota (which, in turn, may yield better or worse welfare outcomes than the fixed quota at \( \bar{q} \)).

Define \( c_t^Q \), \( Q_t^Q \), and \( \tau_t^Q \) as, respectively, the domestic cost, cumulative production, and equivalent tariff at any time \( t \) generated by the quota set at \( \bar{q} \). Further define \( \bar{t}_Q \) as the time when learning ceases under this quota (\( Q_t^Q = \bar{Q} \) at \( \bar{t}_Q \)). Similarly, define \( c_t^\tau \), \( Q_t^\tau \), and \( \bar{t}_\tau \) as the domestic cost, cumulative production, and end learning time for any fixed tariff \( \tau \). I initially assume that the tariff is low enough that its associated learning phase extends past the learning phase associated with the quota (\( \bar{t}_\tau \geq \bar{t}_Q \)). This will imply \( c_t^\tau \geq c_t^Q \) at all times.\(^{39}\) For any such tariff \( \tau \), the generated welfare flow \( \bar{w}^*(c_t^\tau, \bar{c} + \tau) \) will always be below the welfare flow \( \bar{w}^*(c_t^Q, \bar{c} + \tau_t^Q) \) generated by the quota: even if \( c_t^Q \) was higher and equal to \( c_t^\tau \), then the welfare flow under the quota would be higher since it is a maximum for any tariff level at that cost. Furthermore, since \( c_t^Q \) is in fact lower, the welfare flow from the quota at this maximum will be even higher:

\[
\bar{w}^*(c_t^\tau, \bar{c} + \tau) \leq \bar{w}^*(c_t^Q, \bar{c} + \tau) \leq \bar{w}^*(c_t^Q, \bar{c} + \tau_t^Q).
\]

Over the entire planning period, the difference in total welfare between the quota and the tariff will be given by:

\[
TW^Q - TW^\tau = \int_0^T \bar{w}^*(c_t^Q, \bar{c} + \tau_t^Q) - \bar{w}^*(c_t^\tau, \bar{c} + \tau) \, dt,
\]

which will definitely be positive and can be quite large since the integrand will always be non-negative.\(^{40}\)

Finally, there still is the possibility that the welfare flow from a fixed tariff is above the welfare flow from the quota at a point in time when the domestic cost under the tariff is significantly below the domestic cost under the quota. Of course, a high fixed tariff will be required in order to generate

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\(^{39}\)This is due to the fact that the cumulative production paths \( Q_t^Q \) and \( Q_t^\tau \) can cross at only one point in time. \( Q_t^Q \) must be above \( Q_t^\tau \) before they cross which can only happen, by assumption, after \( \bar{t}_Q \). Thus, for any time before \( \bar{t}_Q \), \( c_t^Q \) will be below \( c_t^\tau \). After \( \bar{t}_Q \), \( c_t^Q \) will have attained the lower bound cost \( \bar{c} \).

\(^{40}\)The welfare flows can be equal only for the case of a zero fixed tariff and only after learning under this tariff ceases.
this cost difference. This high tariff would be very unlikely to generate higher total welfare over the entire planning period since it would also entail very high levels of distortion later in the learning phase until the end of the planning period. In this unusual case, the high fixed tariff might yield higher total welfare than the fixed quota at $\tilde{q}$ but would nevertheless still be dominated by a lower (more restrictive) fixed quota set below $\tilde{q}$. In the appendix, it is shown that a fixed quota that generates a learning phase equal in length to that of an arbitrary fixed tariff always yields higher total welfare (both during the learning phase and after) than the tariff.

Small Adjustment Costs

As the exogenous adjustment costs decrease, the social planner can consider changing the level of the trade instrument (tariff or quota) during the planning period. The latter would then be partitioned into smaller periods within which the trade instrument would be fixed. The number of partitions would increase as the adjustment costs fall. In the case of a tariff, the fixed tariff levels over each successive partition would decrease. The last partition would have a zero tariff and extend from the end of the learning phase to the end of the planning period. The number and timing of partitions along with their associated tariff levels would be endogenously determined by the planner as a function of the adjustment costs. In the limit, as the adjustment costs drop to zero, the time path of the optimal tariff will coincide with the optimal flexible tariff path that was previously derived.

In order to evaluate the choice of trade instruments, I assume that the adjustment costs depend only on the number of times the level of the trade instrument is changed. If the chosen trade instrument is a quota, then the optimal policy will similarly involve a partition of the planning period into segments with fixed quota levels. Although the number and timing of the partitions will be different than those chosen under the tariffs, it will nevertheless be possible to compare total welfare levels under the two types of trade instruments. The quotas will continue to dominate the tariffs, again yielding higher total welfare outcomes.

This dominance is driven by a simple consideration: given any individual partition of the planning period and any fixed tariff level within that partition, a fixed quota that generates the same amount of cumulative production as the tariff during this same partition must also yield higher total welfare over this period.\textsuperscript{41} Thus, the outcome of the optimal tariff policy can be

\textsuperscript{41}See the appendix for a proof.
easily compared to a quota policy that uses the same partitions and induces the same amount of cumulative domestic production over each separate partition (the fixed quota levels over each partition are set in order to maintain this equality). This quota policy will clearly dominate the optimal tariff policy as it will yield higher total welfare in every partition while incurring the same amount of adjustment costs. The optimal quota policy will yield even further gains as the planner optimally chooses the number and timing of partitions for the quota.

The Mill-Bastable Test

If we now consider the cases where the initial cost is high enough to preclude demand for the domestic good under free trade, then the option of no protection (and thus no domestic production) is potentially optimal. Once again, the social planner needs to trade off the welfare flow benefits of protection against the fixed learning cost. These tradeoffs will be similar to the case of the fully flexible tariff, except that the welfare flow benefits will be lower as they are additionally traded-off against the adjustment costs.

8 Additional Advantages of the Quota

When the high initial domestic cost precludes demand for the domestic good under no protection and the planning period extends significantly past the end of the learning phase, then the choice of protection with the fixed quota (at $\bar{q}$) will clearly be preferred to no protection: the fixed learning cost can be repaid, not only by higher welfare flows during the learning phase, but also by the welfare flows at the unrestricted maximum of $\overline{W}$ which will accrue from the end of the learning phase until the end of the planning period. If the adjustment costs are also high and preclude changing the level of the trade instrument, then the fixed tariff will offer a terrible alternative to the fixed quota: in order to induce initial production of the domestic good, the tariff would have to be set at a very high level comparable to the level of the initial equivalent tariff associated with the quota. Once learning progresses, this high tariff will create ever increasing distortions. When learning ceases, this high level of distortion (which will generate a welfare flow far below $\overline{W}$) will continue until the end of the planning period.

This considerable difference between the welfare effects of the quota and tariff could potentially outweigh any amount of revenue loss due to practical considerations involved with the administra-

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42 The domestic cost at the beginning and end of every partition will be the same under both policies since the same amount of cumulative production occurs over each partition.
tion of a quota. Thus, even a voluntary export restraint (assuming that the domestic country’s transfer of revenue to foreign suppliers is not politically necessary to enact the restraint) could yield higher welfare gains to the domestic country than any fixed tariff alternative.

Although a fixed subsidy instrument is not formally modeled in this paper, it is also clearly possible for the fixed quota to yield higher welfare gains than an optimally chosen fixed subsidy. Even though the subsidy does not generate any consumption distortions of the foreign good (as do the quota and tariff), the rigidity of the subsidy would nevertheless create the same types of problems as those mentioned for the tariff: a fixed subsidy will either not protect the infant industry enough early in the learning phase, or it will protect it too much later on.

Finally, the fixed quota will exhibit one other advantage to policy makers over both the tariffs and subsidies, even when these were fully flexible: a lower information requirement for implementation. In order to calculate the long run consumption level of the foreign good \( \bar{q} \) (and hence the optimal fixed quota), a policy maker only needs information on the foreign cost, the lower bound domestic cost, and demand conditions. In particular, no information on the shape of the learning curve (including its duration) is required. On the other hand, the setting of the optimal subsidies (when feasible) or tariffs, even when these instruments are fully flexible, requires this learning curve information. The learning curve may be known to the firms and not to the policy maker, in which case the firms would have a strong incentive to distort any current and future cost information collected by the government.\(^{43}\) Furthermore, the learning curve may also have a stochastic element that is also unknown to the firms. Although the uncertainty is not formally modeled in this paper, the presence of learning curve uncertainty can only reduce the effectiveness of the optimal subsidies or tariffs while it does not affect the performance of the optimal quota.\(^{44}\)

9 Conclusion

This paper has focused on the practical considerations involved in policy decisions for infant industry protection. A policy maker first wants to make sure that a candidate industry only needs temporary protection and that this protection will generate higher cumulative benefits than costs. The paper

\(^{43}\)This problem was studied in Dinopoulos, Lewis and Sappington (1995). Their results show that the presence of asymmetric learning curve information between firms and the government may preclude protection that would have been optimal under symmetric information. In general, even asymmetric information about current costs (and not future costs) may prevent the government from enacting trade policies which would index the tariff or subsidy to the difference between current and lower bound cost.

\(^{44}\)I am assuming uncertainty about the shape and duration of the learning curve, and not uncertainty about the lower bound cost, which would affect the design of the optimal quota.
shows how this Mill-Bastable Test can be re-formulated in a way that makes it easier to apply (when the effects of discounting are negligible): the cumulative costs can be approximated by a fixed learning cost that only depends on the learning curve. If the uses of different policy instruments meet the conditions of the Mill-Bastable Test, the policy maker must then choose the optimal policy instrument for protection. The paper shows how limitations on the instrument’s flexibility over time strongly affect this choice. Ideally, the policy maker wants to decrease the level of protection as learning progresses and eliminate protection once learning has ceased. Subsidies or tariffs need to be constantly lowered over time to produce this effect; these adjustments may not be feasible in practice. A fixed quota, on the other hand, automatically reduces its level of protection as domestic costs fall. The fixed quota can also be chosen so as to become non-binding once learning ceases. These characteristics endow the quota with advantages over the tariff and subsidy. Any uncertainty concerning the learning curve re-enforces these advantages.

The quota, however, also has some well known drawbacks vis-a-vis the tariff or subsidy. Quotas, even when their rights are auctioned, typically generate less revenue than comparable tariffs. They also, like tariffs, distort consumption decisions whereas domestic production subsidies do not. This paper does not intend to minimize these drawbacks but rather emphasizes the quota’s particular advantages that specifically pertain to infant industry protection. The paper shows how, in this context, the advantages of the quota (especially over the tariff) are quite significant and can realistically outweigh these better known disadvantages. This paper does not intend to defend the use of quantity restrictions for infant industry protection as necessarily sound economic policy. Instead, this paper suggests that the use of these quantity restrictions is less distorting than has been previously considered and that it may be, in some specific cases, the outcome of welfare maximizing behavior by the government. Furthermore, recommendations for future infant industry policies must also consider the problems that costly policy adjustments and learning curve uncertainty create for tariffs and subsidies. In certain cases, a policy maker will be confident that the industry’s cost will drop over time but will be equally confident that any protection policy, once implemented, will be hard or impossible to repeal. In these cases, a quota will offer an attractive policy instrument that will ensure that the protection it provides will be temporary and will decrease with the changing domestic costs.
Appendix

A The Effects of Discounting on Total Welfare Comparisons

Some derivations in this paper assumed a negligible discount rate in order to simplify the comparison of total welfare levels under different policy scenarios. Given any of the trade instrument restrictions considered in this paper, an increase in the exogenous discount rate will affect the optimal protection levels by making protection less valuable.\(^{45}\) On the other hand, any reasonably low discount rate will not affect any of the total welfare comparisons derived earlier.

When the effects of discounting cannot be ignored, total welfare \(TW\) over the planning period can still be decomposed into the accumulated welfare flows \(\bar{w}(q_t, \tilde{q}_t)\) and the fixed learning cost \(FLC\) in the following way:

\[
TW = \int_0^T e^{-\rho t} W_t \, dt
\]

\[
= \int_0^T e^{-\rho t} \left[ U(q_t, \tilde{q}_t) - \bar{c}_t q_t - \tilde{c}_t \tilde{q}_t \right] dt - \int_0^T e^{-\rho t} \left[ (c_t - \bar{c}) q_t \right] dt
\]

\[
= \int_0^T e^{-\rho t} \left[ U(q_t, \tilde{q}_t) - \tilde{c}_t q_t - \tilde{c}_t \tilde{q}_t \right] dt - \alpha \cdot \int_0^{\bar{Q}} (c(Q) - \bar{c}) \, dQ
\]

\[
= \int_0^T e^{-\rho t} \bar{w}(q_t, \tilde{q}_t) \, dt - \alpha \cdot FLC,
\]

where \((q_t, \tilde{q}_t)\) is any consumption path such that learning occurs and ends at a time \(\bar{t}\) before the end of the planning period \(T\), and \(\alpha\) is some number between \(e^{-\rho \bar{t}}\) and 1.\(^{46}\) Similarly, when the domestic good is not produced, total welfare would then be written:

\[
TW^o = \int_0^T e^{-\rho t} \bar{w}(0, q^o(\bar{c})) \, dt = \int_0^T e^{-\rho t} W^o \, dt.
\]

Optimal Protection Versus no Protection

Given a positive level of domestic production, let \((q^*_t, \tilde{q}^*_t)\) be the optimal consumption paths under protection (the trade instruments used to protect may be restricted in the ways previously discussed.) The decision to protect the domestic industry then depends on the comparison of the total welfare levels \(TW^* = \int_0^T e^{-\rho t} \bar{w}(q^*_t, \tilde{q}^*_t) \, dt - \alpha \cdot FLC\) and \(TW^o = \int_0^T e^{-\rho t} W^o \, dt\). Higher welfare flows \(\bar{w}(q^*_t, \tilde{q}^*_t) \geq W^o\) must again be weighed against the learning cost \(\alpha \cdot FLC\).

\(^{45}\)As the discount rate approaches infinity, no protection would always be optimal.

\(^{46}\)\(\alpha\) will depend on both the discount rate \(\rho\) and the consumption paths \((q, \tilde{q})\).

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Comparative statics involving the level of product differentiation will affect the difference between \( \bar{w}(q^*_t, \tilde{q}^*_t) \) and \( W^o \) in the same way as was previously derived. Since this comparison involves the flows at each point in time, the discounting of these flows will not affect the comparisons. Similarly, comparative statics involving the domestic cost bounds and shape of the learning curve will affect \( FLC \) in an identical way as was previously derived. The only problem will be that these comparative statics will also affect the level of \( \alpha \). Given a small discount rate and a relatively short learning phase, this change in \( \alpha \) will be dominated by the changes in the welfare flows or the learning cost \( FLC \). Thus, the effects of product differentiation, the length of the planning period, cost bounds, and shape of the learning curve on the choice to protect the infant industry will remain unchanged under any reasonable discounting.

**Fixed Tariff Versus Fixed Quota**

It was shown earlier that, given certain conditions on the shape of the utility function and the shape of the learning curve, the welfare flow \( \bar{w}(q_t, \tilde{q}_t) \) generated by the fixed quota at \( \bar{q} \) is always higher than the welfare flow generated by any fixed tariff. The discounting of these flows in the total welfare computation will not affect the total welfare rankings of the quota and tariff: later differences in the flows will just count less than earlier differences, but these differences will still always have the same sign favoring the quota. On the other hand, the values of \( \alpha \) entering into the total welfare computation will now be slightly different under the quota and the tariff. The learning cost \( \alpha \cdot FLC \) under the two instruments will no longer be identical and will not cancel each other out in the total welfare comparisons. Again, these changes in \( \alpha \) will be small and will be dominated by the difference in the welfare flows. The comparison of total welfare under the two instruments will be very unlikely to be reversed by the inclusion of these discounting effects.

**B Extensions of Total Welfare Comparisons Between a Fixed Tariff and Quota**

**Comparisons Over a Partition of the Planning Period**

Assuming negligible discounting effects and quadratic utility, I show that the total welfare generated by any fixed tariff over any partition of the planning period is necessarily lower than the total welfare generated over the same period by an appropriately chosen quota. Let \( \tau \) be any fixed tariff level over any partition of the planning period extending from time \( t_1 \) to \( t_2 \). Cumulative production is
initially $Q_1$ at time $t_1$ and increases to $Q_2$ at $t_2$. Given any consumption paths $(q_t, \tilde{q}_t)$ such that $Q(t_1) = Q_1$ and $Q(t_2) = Q_2$, total welfare during the partition can again be decomposed into a sum of welfare flows that do not depend on the changing domestic cost and a separate fixed learning cost:

$$TW = \int_{t_1}^{t_2} W_t dt = \int_{t_1}^{t_2} \left[ U(q_t, \tilde{q}_t) - \hat{c}q_t - \tilde{c}\tilde{q}_t \right] dt - \int_{Q_1}^{Q_2} \left[ c(Q) - \hat{c} \right] dQ$$

$$= \int_{t_1}^{t_2} \hat{w}(q_t, \tilde{q}_t) dt - \hat{FLC},$$

where $\hat{c}$ is any arbitrary cost level. Thus, given any two consumption paths that generate the same cumulative production during any given time period, the difference in total welfare generated by these two paths will be given by the accumulated difference in the welfare flow $\hat{w}(q_t, \tilde{q}_t)$:

$$\Delta TW = \int_{t_1}^{t_2} \Delta \hat{w}(q_t, \tilde{q}_t) dt.$$ 

Again, define $\hat{w}^*(p_t, \tilde{p}_t) = \hat{w}\left(q(p_t, \tilde{p}_t), \tilde{q}(p_t, \tilde{p}_t)\right)$ as the new welfare flow measured as a function of prices instead of quantities and let $\tau^*(c_t)$ now represent the tariff level that maximizes $\hat{w}^*(c_t, \hat{c} + \tau)$ given $c_t$. Thus, given any fixed quota level, a choice of $\hat{c}$ such that $\tilde{q}(\hat{c}, \tilde{c})$ is equal to this quota level will ensure that $\tau^*(c_t)$ will always be equal to the equivalent tariff generated by the quota. In other words, this choice of $\hat{c}$ guarantees that the tariff level that maximizes the welfare flow $\hat{w}^*(c_t, \hat{c} + \tau)$ given $c_t$ will be precisely the tariff that induces consumption of the foreign good at the quota level.

The total welfare induced by the fixed tariff can now be compared to that induced by a particular fixed quota. The fixed quota level is chosen such that the cumulative production generated during the partition is exactly equal to that generated by the tariff. $\hat{c}$ is then chosen as described above. Let $c_t^Q$ and $\tau_t^Q$ denote the domestic cost and equivalent tariff generated by this quota between $t_1$ and $t_2$. Similarly, let $c_t^\tau$ denote the domestic cost generated by the fixed tariff $\tau$. The total welfare difference between the tariff and quota over the partition will be given by:

$$TW^Q - TW^\tau = \int_{t_1}^{t_2} \hat{w}^*(c_t^Q, \hat{c} + \tau_t^Q) - \hat{w}^*(c_t^\tau, \hat{c} + \tau) dt$$

This difference must be positive since the welfare flow difference at any time during the time period will be positive. The welfare flow under the quota at any point in time will be higher for two
reasons:

\[ \tilde{w}^*(c_t^q, \tilde{c} + \tau) \leq \tilde{w}^*(c_t^q, \tilde{c} + \tau^*(c_t^q)) \leq \tilde{w}^*(c_t^q, \tilde{c} + \tau_t^Q) \]

The first inequality reflects the fact that the quota yields a maximum of the welfare flow function at any given domestic cost level. The second inequality is driven by the fact that the domestic cost under the quota \((c_t^Q)\) is below the domestic cost under the tariff \((c_t^\tau)\) throughout the time period.  

**More Flexible Forms of the Utility Function**

If utility is quadratic (the demand functions are linear) then, given \(c_t\), the tariff level \(\tau^*(c_t)\) that maximizes the welfare flow \(\tilde{w}^*(c_t, \tilde{c} + \tau)\) is always equal to the equivalent tariff \(\tau_t^Q\) generated by a fixed quota set at \(\tilde{q}\) (as was shown in the paper). If the demand system is non-linear, then the equivalent tariff \(\tau_t^Q\) will be close, but not equal to \(\tau^*(c_t)\). The difference between \(\tau_t^Q\) and \(\tau^*(c_t)\) will increase with the curvature of the level sets of \(\tilde{q}(p, \tilde{p})\). This will make it harder to compare the welfare flows generated by the fixed tariff and quota at all points in time. Nevertheless, all of the previous reasons explaining the superiority of the quota over the tariff will remain valid with non-quadratic utility.

Certain more general properties of the welfare flows \(\tilde{w}^*(c_t, \tilde{c} + \tau)\) can be obtained if this function is assumed to be concave. This will be the case as long as the demand system is not too convex. There will then be a unique tariff level \(\tau^*(c_t)\) that maximizes \(\tilde{w}^*(c_t, \tilde{c} + \tau)\) for any \(c_t\). Given any \(c_t\), this welfare flow will be increasing with the tariff level for any tariff below \(\tau^*(c_t)\) and will be decreasing with the tariff level when the tariff is above \(\tau^*(c_t)\). \(\tau^*(c_t)\) will be decreasing with \(c_t\) while the welfare flow at this maximum increases. The difference between the equivalent tariff level \(\tau_t^Q\) and \(\tau^*(c_t)\) will also decrease with \(c_t\). Thus, under the fixed quota, the welfare flows \(\tilde{w}^*(c_t, \tilde{c} + \tau_t^Q)\) will increase as learning progresses, reaching the unrestricted maximum level of \(\tilde{W}\) when learning ceases.

A fixed tariff will create the same unnecessary distortions as were previously described: it will either not protect enough early on or protect too much later. As the fixed tariff level increases from zero, the maximum of the generated welfare flow \(\tilde{w}^*(c_t, \tilde{c} + \tau)\) over time will steadily decrease from \(\tilde{W}\). Furthermore, this lower maximum level will not be sustained once it is reached during the

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47The cumulative production paths under a fixed quota and tariff can cross at only one point in time. By construction, this point happens at the end of the time period at \(t_2\). Cumulative production under the quota will therefore be higher (and domestic cost lower) than under the tariff at any time before \(t_2\).
learning phase. The welfare flows will decrease after this point for the remaining of the learning phase. Once learning ceases, the flows will remain constant at this lower level for the remaining of the planning period. The value of this post learning constant welfare flow will also steadily decrease as the fixed tariff level is increased.

Although it is now possible for the welfare flow under a fixed tariff to exceed that from the fixed quota at a given point in time, this possibility is nevertheless remote and can only happen for a limited time span. It would be even more unlikely for the accumulated total welfare during the entire planning period under the tariff to exceed the total welfare accumulated under the quota.

References


