Procrastination on Long-Term Projects

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March 27, 2001

Abstract

Whereas previous papers on time-inconsistent procrastination assume that projects are completed once begun, we develop a model of long-term projects in which a person chooses when to complete each stage, where the effort costs for the different stages may vary. In addition to procrastination in starting a valuable project, a naive person might undertake costly effort to begin a project but then never complete it. When the costs of completing different stages are more unequal, procrastination is more likely, and it is when the costs are larger for later stages that people start but don’t finish projects. Moreover, we show that if the structure of costs over the course of a project is endogenous, people may be prone to choose a cost structure that leads them to start but not finish that project. We extend the model to the case where a person can complete multiple projects in succession and to the case where a person must pay a small cost to keep open the option of completing the project in the future.

Keywords: Choice, Long-Term Projects, Naivete, Partial Naivete, Present-biased Preferences, Procrastination, Self Control, Sophistication, Time Inconsistency.

JEL Category: A12, B49, C70, D11, D60, D74, D91, E21

Acknowledgments: For financial support, we thank the National Science Foundation (Awards SBR-9709485 and SES-0078796), and Rabin thanks the Russell Sage and MacArthur Foundations.

1. Introduction

There is a growing literature in economics that assumes people have self-control problems, conceived of as a time-inconsistent taste for immediate gratification, and an often discussed implication of such preferences is procrastination.\(^1\) Most of this research on procrastination assumes that a “project” requires a single period of effort.\(^2\) But most real-world projects, in contrast, take some duration to complete, and the effort costs typically vary for different stages of the project.

In this paper we develop a simple model of long-term projects, and analyze procrastination in this environment. We describe how the structure of costs over the course of a project plays an important role in whether a person delays completing that project. We show that, in addition to procrastination in starting a beneficial project, a person might start a project but then procrastinate finishing that project. Moreover, we show that if the structure of costs over the course of a project is endogenous, people may be prone to choose a cost structure that makes it very likely that they start but do not finish that project.

In Section 2, we describe a formalization of time-inconsistent preferences originally developed by Phelps and Pollak (1968) in the context of intergenerational altruism, and later employed by Laibson (1994) to capture self-control problems within an individual: In addition to time-consistent discounting, a person always gives extra weight to current well-being over future well-being. These “present-biased preferences” imply that each period a person tends to pursue immediate gratification more than she would have preferred if asked in any prior period. We also discuss in Section 2 an important issue that arises when people have self-control problems: How aware is a person of her future self-control problems? The results in this paper, as in previous papers, emphasize the role that naivety — being (at least somewhat) unaware of future self-control problems — plays in procrastination. Towards this end, our analysis compares four types of people: TCs have standard time-consistent preferences, sophisticates have self-control problems and are fully aware of those self-control problems, naifs have self-control problems and are fully unaware of those self-control problems, and partial naifs have self-control problems, are aware of those self-control problems, but underestimate their magnitude.

In Section 3, we present our model of long-term projects. We assume for simplicity that a project

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\(^1\) For recent papers discussing procrastination, see, for instance, Prelec (1989), Akerlof (1991), Fischer (1997), and O’Donoghue and Rabin (1999a, 1999b, 1999c, forthcoming).

\(^2\) A notable exception is Fischer (1997), who studies the behavior of sophisticates who must put in a fixed amount of time before some deadline.
has two stages, which we often refer to as “starting” and “finishing” the project. There is an infinite number of periods in which the person can work on the project, and in each period the person can take one of two actions: She can either complete the current stage, in which case she incurs an immediate cost associated with that stage, or do nothing. Completion of the first stage does not generate any benefits, but when (and if) the person completes the second stage, she initiates an infinite stream of benefits that begin the following period. In Section 3, we take the structure of costs over the course of the project to be exogenous.

TCs immediately start and then finish the project if and only if the project is worth doing in terms of its net discounted present value. Sophisticates might not begin the project because, given their taste for immediate gratification, the project is not worth doing. Sophisticates also might not begin the project because they realize that when the second-stage cost becomes immediate they will no longer find it worth incurring. Hence, sophisticates, like TCs, never begin a project without completing it. But unlike TCs — for whom the intertemporal structure of costs does not matter — whether sophisticates complete the project can depend on the intertemporal structure of the costs, where a higher second-stage cost can lead them not to complete the project even when they would complete the project the costs were more evenly distributed.

Our analysis shows that, in addition to the reluctance that sophisticates have to starting a project, people who are (at least partially) naive may procrastinate, persistently planning to work on the project, but perpetually putting this work off. As in previous analyses of procrastination, a person might never start a valuable project. But an even more costly form of procrastination can arise for long-term projects, however, because a person might start a project that she expects to finish but then never finishes. In this case, the cost of procrastination is not just the foregone benefits from a valuable project, but also the wasted effort incurred to start the project that never produces any benefits. Indeed, we show that for any taste for immediate gratification and for any degree of naivete, no matter how large the cost of starting the project, there exist combinations of benefits and second-stage costs that lead the person to undertake the first stage but never complete the project.

We also show in Section 3 that whether and how a person procrastinates depends crucially on the structure of costs over the course of the project. There are two ways in which structure matters: the allocation and the order. In terms of allocation, we show that, for a fixed total cost, procrastination is least likely when costs are allocated evenly across stages. Whether a person procrastinates a given stage depends on a comparison between the cost of completing that stage, which is what
the person wants to put off, and the forgone benefits from such a delay. The key question that determines whether the person completes the project is whether she procrastinates the highest-cost stage. In terms of order, we show that procrastination is more costly if the lower-cost stage comes first, because if a person is going to procrastinate the high-cost stage, it’s better that she not first complete the low-cost stage. In other words, it’s when the low-cost stage comes first that the new and more costly form of procrastination can arise.

Our analysis in Section 3 shows how the structure of costs over the course of the project is an important determinant of procrastination. In Section 4, we endogenize this cost structure. We consider two cases. First, we suppose the costs of the two stages are fixed, but we permit the person to choose the order in which she completes the stages. Second, we permit the person to choose the allocation of costs over the course of the project — e.g., to choose between an even allocation vs. an uneven allocation. Because the same preference for immediate gratification that leads to procrastination also leads the person to prefer incurring smaller costs now and larger costs in the future, when given the option the person chooses to do the lower-cost stage first and chooses unequal allocations. As a result, the person is prone to choose a cost structure that maximizes the likelihood that she will start the project but not finish it. Hence, it is not only possible that a new and costlier form of procrastination might arise for long-term projects, but there are in fact forces that make it likely that this new form of procrastination will indeed arise.³

In Section 5, we consider two extensions of our model to richer environments where procrastination may cause a person to incur other unnecessary costs. First, we suppose that a person has multiple long-term projects to complete. While having multiple projects can help motivate a person not to procrastinate because delay of the current project imposes delay on future projects, we show that a new distortion can arise: If starting a new project is less onerous than finishing an already started project, then a person may be too prone to start new projects. Second, we suppose that if the person chooses to delay, she must pay a small cost to keep open the option of completing the project in the future. Such an option cost of course helps motivate the person not to procrastinate. But if she does procrastinate, she may pay the option cost over and over again, persistently planning to eventually complete the project, when in fact she may never get around to it.

Section 6 concludes.

³ By showing that adding the option of how to structure the project can make a naive procrastinator worse off, the results in Section 4 are similar to those in O’Donoghue and Rabin (forthcoming) where providing a person with additional options can make a person more prone to procrastinate.
2. Present-Biased Preferences

The standard economics model assumes that intertemporal preferences are *time-consistent*: A person’s relative preference for well-being at an earlier date over a later date is the same no matter when she is asked. But there is a mass of evidence that intertemporal preferences take on a specific form of *time inconsistency*: A person’s relative preference for well-being at an earlier date over a later date gets stronger as the earlier date gets closer. In other words, people have self-control problems caused by a tendency to pursue immediate gratification in a way that their “long-run selves” do not appreciate.4

In this paper, we apply a simple form of such *present-biased preferences*, using a model originally developed by Phelps and Pollak (1968) in the context of intergenerational altruism and later used by Laibson (1994) to model time inconsistency within an individual.5 Let $u_t$ be the instantaneous utility a person gets in period $t$. Then her intertemporal preferences at time $t$, $U^t$, can be represented by the following utility function:

$$U^t(u_t, u_{t+1}, ..., u_T) \equiv \delta^t u_t + \beta \sum_{\tau=t+1}^{T} \delta^\tau u_\tau.$$

This two-parameter model is a simple modification of the standard one-parameter, exponential-discounting model. The parameter $\delta$ represents standard “time-consistent” impatience, whereas the parameter $\beta$ represents a time-inconsistent preference for immediate gratification. For $\beta = 1$, these preferences are time-consistent. But for $\beta < 1$, at any given moment the person has an extra bias for now over the future.

To examine intertemporal choice given time-inconsistent preferences, one must ask what a person believes about her own future behavior. Most of the literature has focused on two extreme assumptions: *Sophisticated* people are fully aware of their future self-control problems and therefore correctly predict how their future selves will behave, and *naive* people are fully un-aware of their future self-control problems and therefore believe their future selves will behave exactly as

\[4\] See, for instance, Ainslie (1975, 1991, 1992), Ainslie and Haslam (1992a, 1992b), Loewenstein and Prelec (1992), Thaler (1991), and Thaler and Loewenstein (1992). While the rubric of “hyperbolic discounting” is often used to describe such preferences, the qualitative feature of the time inconsistency is more general, and more generally supported by empirical evidence, than the specific hyperbolic functional form.

\[5\] This model has since been used by Laibson (1996, 1997), Laibson, Repetto, and Tobacman (1998), O’Donoghue and Rabin (1999a, 1999b, 1999c, forthcoming), Fischer (1997), and others.
they currently would like them to behave.\(^6\) Recently (O’Donoghue and Rabin (forthcoming)), we have formulated an approach to the more realistic assumption *partial naivete* wherein a person is aware that she will have future self-control problems but underestimates their magnitude. We suppose that a person has true self-control problem \(\beta\), but perceives that in the future she will have self-control problem \(\hat{\beta}\). In other words, in any given period the person’s current preferences are characterized by \(\beta\), but she perceives that in the future she will behave like a sophisticated person with preferences characterized by \(\hat{\beta}\). With this formulation, people with standard time-consistent preferences — whom we refer to as TCs — have \(\hat{\beta} = \beta = 1\), sophisticates have \(\hat{\beta} = \beta < 1\), naifs have \(\beta < \hat{\beta} = 1\), and partial naifs have \(\beta < \hat{\beta} < 1\).\(^7\)

Our focus in this paper is how naivete about future self-control problems can lead to procrastination on long-term projects. Much of our analysis will focus on people who are completely naive, for whom our results are strongest. But since we wish to emphasize that the intuitions we identify apply even for people who are only partially naive, we also derive results for partial naifs. In the next section, we shall define a formal solution concept — that applies to sophisticates, naifs, partial naifs, and TCs — within our specific model.

### 3. Model with Exogenous Cost Structure

For most of our analysis, we focus for simplicity on two-stage projects; we discuss in Section 6 how our lessons extend to longer projects. A *long-term project* consists of two stages, and completing each stage is onerous in the sense that completing it requires that the person incur an immediate cost. In this section, we assume that the cost structure is exogenous, where the first stage requires cost \(c > 0\) and the second stage requires cost \(k > 0\). We endogenize this structure in Section 4.

\(^6\) Strotz (1956) and Pollak (1968) carefully lay out these two assumptions (and develop the labels), but do not much consider the implications of assuming one versus the other. Fischer (1997) and Laibson (1994, 1996, 1997) assume sophisticated beliefs. O’Donoghue and Rabin (1999a) consider both, and explicitly contrast the two.

\(^7\) For simplicity, we abstract away from some complications that might arise with partial naivete. First, we assume a person is absolutely positive — though wrong when \(\hat{\beta} > \beta\) — about her future self-control problems. We doubt that our qualitative results would change much if the person had probabilistic beliefs whose mean underestimated the actual self-control problem. But it is central to our analysis that a person not fully learn over time her true self-control problem, or, if she does come to recognize her general self-control problem, she still continues to underestimate it on a case-by-case basis. Second, we assume that all higher-order beliefs — e.g., beliefs about future beliefs — are also equal to \(\hat{\beta}\). Hence, a person has what might be called “complete naivete about her naivete”: A partially-naive person thinks she will be entirely aware in the future of what she now believes is the extent of her future self-control problems (since otherwise she predicts she will forget what she currently knows). While alternatives are not without merit — we suspect that people do sometimes realize that they are too often over-optimistic — we think our modeling choice here is the most realistic and most tractable.
A person carries out a long-term project because of the future benefits it creates. For simplicity, we assume that the person must complete both stages before she can reap any benefits; we discuss in Section 6 how our results extend to the case where some of the benefits start accruing upon partial completion. We assume that completion of stage 2 in period \( \tau \) initiates a stream of benefits \( v \geq 0 \) in each period from \( \tau + 1 \) onward.\(^8\)

We assume there is an infinite number of periods in which the person can work on the project, and in each period the person can take one of two actions: She can complete the current stage or do nothing. Hence, in any period before which the person has not yet completed anything, she can choose either to do nothing or to complete the first stage; and in any period before which she has completed the first stage, she can choose either to do nothing or to complete the project.

Our solution concept, “perception-perfect strategies”, requires that at all times a person have reasonable beliefs about how she would behave in the future following any possible current action, and that she choose her current action to maximize her current preferences given these beliefs. Perception-perfect strategies depend on the two attributes of a person discussed in Section 2 — her self-control problem \( \beta \), and her perceptions of future self-control problems \( \hat{\beta} \). We now define a formal solution concept within our specific model.

Let \( A \equiv \{0, 1\} \) be the set of actions available in each period, where \( a = 0 \) means “do nothing” and \( a = 1 \) means “complete the current stage”. Let \( h^t \in \{\emptyset, 1, 2, ..., t - 1\} \) be a history in period \( t \), where \( h^t = \emptyset \) means the person has not completed stage 1 prior to period \( t \), and \( h^t = \tau \in \{1, 2, ..., t - 1\} \) means the person completed stage 1 in period \( \tau \). A strategy is a function \( s \) such that if the history in period \( t \) is \( h^t \), then strategy \( s \) specifies action \( s(h^t, t) \in \{0, 1\} \). In the usual game-theoretic sense, a strategy is a plan for what to do in all possible contingencies. But we shall use strategies to represent both a person’s beliefs about future behavior and her true behavior, which may differ when \( \hat{\beta} \neq \beta \).

Let \( V^t(a_t, h^t, s, \beta) \) represent the person’s period-\( t \) preferences over current actions given history \( h^t \) and conditional on following strategy \( s \) beginning in period \( t + 1 \). Then:

\(^8\) Hence, our formal assumption is that if the person completes stage 1 in period \( a \) and stage 2 in period \( b > a \), then her instantaneous utilities are \( u_a = -c, u_b = -k, u_\tau = v \) for all \( \tau \in \{b + 1, b + 2, \ldots\} \), and \( u_\tau = 0 \) otherwise. The crucial feature of a procrastinatory environment is that costs are immediate whereas benefits are delayed.
\[
V_t(a, h^t, s, \beta) \equiv \begin{cases} 
-c - \beta \delta^d k + \frac{\beta \delta^{d+1}}{1-\delta} v & \text{if } h^t = \emptyset, a_t = 1, \text{ and} \\
-\beta \delta^d c - \beta \delta^d k + \frac{\beta \delta^{d+1}}{1-\delta} v & \text{if } h^t = \emptyset, a_t = 0, \\
-k + \frac{\beta \delta}{1-\delta} v & \text{if } h^t = \tau \neq \emptyset, \text{ and } a_t = 1 \\
-\beta \delta^d k + \frac{\beta \delta^{d+1}}{1-\delta} v & \text{if } h^t = \tau \neq \emptyset, a_t = 0, \text{ and} \\
& d \equiv \min\{x > 0 | s(t, t + x) = 1\} \\
& d' \equiv \min\{x > d | s(t + d, t + x) = 1\} \\
& d \equiv \min\{x > 0 | s(\tau, t + x) = 1\}.
\end{cases}
\]

The four cases in this equation correspond to four different possibilities of when, relative to period \( t \), the person completes the two stages. In the first case, the person completes the first stage now and the second stage in the future. In the second case, the person completes both the first stage and the second stage in the future. In the third case, the person has completed the first stage in the past (in period \( \tau < t \)) and completes the second stage now. In the fourth case, the person has completed the first stage in the past (in period \( \tau < t \)) and completes the second stage in the future.

With this notation, we can provide a formal definition of perception-perfect strategy:

**Definition 1.** Given \( \hat{\beta} \), strategy \( \hat{s} \) represents \( \hat{\beta} \)-consistent beliefs if for all \( t \) and \( h^t \),

\[
\hat{s}(h^t, t) = \arg \max_{a \in \{0, 1\}} V_t(a, h^t, \hat{s}, \hat{\beta}).
\]

Given \( \beta \) and \( \hat{\beta} \), strategy \( s \) is a perception-perfect strategy if there exists \( \hat{\beta} \)-consistent beliefs \( \hat{s} \) such that for all \( t \) and \( h^t \),

\[
s(h^t, t) = \arg \max_{a \in \{0, 1\}} V_t(a, h^t, \hat{s}, \beta).
\]

A perception-perfect strategy represents how a person with self-control problem \( \beta \) and perceptions of future self-control problems \( \hat{\beta} \) would actually behave in all contingencies. The first component of a perception-perfect strategy is that the person must have some beliefs \( \hat{s} \) for how she would behave in the future, and then in any situation she must choose an optimal action given these beliefs and her current preferences (which depend on \( \beta \)).

The second component of a perception-perfect strategy is that the beliefs \( \hat{s} \) must be “consistent” with the person’s perception of future self-control problems \( \hat{\beta} \). Definition 1 imposes two aspects of consistency, which O’Donoghue and Rabin (forthcoming) refer to as internal consistency and

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9 Throughout we assume for simplicity that when a person is indifferent between \( a = 0 \) and \( a = 1 \), she chooses \( a = 1 \).
external consistency. Internal consistency requires beliefs that for all possible contingencies specify an action that is optimal given her beliefs for subsequent periods and her preferences at the time (which depend on $\hat{\beta}$ and not $\beta$). Internal consistency implies that at all times the person perceives that in the future she will behave like a sophisticated person with self-control problem $\hat{\beta}$. External consistency requires that the person have the same beliefs across contingencies in the sense that for all $t < \tau$ she must have the same belief for what she would do in period $\tau$ following history $h^\tau$ — that is, $\hat{s}(h^\tau, \tau)$ is independent of $t$. This restriction rules out procrastination arising from a form of irrational expectations that goes beyond merely mispredicting self-control, because without it the person could repeatedly reconstruct beliefs that permit her to delay.\footnote{Of course, the restriction of external consistency matters only if there are multiple $\hat{\beta}$-consistent beliefs. The restrictions imposed by external consistency essentially correspond to the additional restrictions which subgame-perfect equilibrium imposes beyond non-equilibrium backwards induction. By the same token, these restrictions would be unnecessary in generic, finite-period situations where “perceptual backwards induction” would yield a unique prediction.}

In words, a perception-perfect strategy reflects that a person perceives that in the future she will behave like a sophisticated person with self-control problem $\hat{\beta}$, and then she chooses her current behavior given these beliefs about future behavior and given her current self-control problem $\beta$. With our formulation, TCs have $\beta = \hat{\beta} = 1$, sophisticates have $\beta = \hat{\beta} < 1$, naifs have $\beta < \hat{\beta} = 1$, and partial naifs have $\beta < \hat{\beta} < 1$. For $\hat{\beta} = 1$ — that is, for TCs and naifs — there is a unique set of $\hat{\beta}$-consistent beliefs and therefore a unique perception-perfect strategy (which we prove in Lemma 2 below). For $\hat{\beta} < 1$ — that is, for sophisticates and partial naifs — there can exist multiple sets of $\hat{\beta}$-consistent beliefs, and therefore there can exist multiple perception-perfect strategies. For simplicity, our main analysis focuses on perception-perfect strategies with “optimistic beliefs”, which means that the person believes either that she will complete stage 2 immediately after completing stage 1 or that she will never complete stage 2.\footnote{For $\hat{\beta} = 1$, the unique set of $\hat{\beta}$-consistent beliefs are indeed optimistic.} Formally:

**Definition 2.** $\hat{\beta}$-consistent beliefs $\hat{s}$ are optimistic if for all $\tau \in \{1, 2, \ldots\}$ either $\hat{s}(\tau, \tau + 1) = 1$ or $\hat{s}(\tau, \tau') = 0$ for all $\tau' > \tau$.

Optimistic beliefs always exist, and by restricting attention to optimistic beliefs (for sophisticates and partial naifs), we get uniqueness in how many stages are completed, although not uniqueness in when they are completed (which we also prove in Lemma 2 below). There also exist perception-perfect strategies with non-optimistic beliefs, however, and these perception-perfect strategies can involve different patterns of observed behavior. For instance, there might be a perception-perfect
strategy with optimistic beliefs under which the person (eventually) completes the project while for the same parameters there exists a perception-perfect strategy with non-optimistic beliefs under which the person never even starts the project. We discuss such issues in Appendix A, including how some difficulties disappear in the $\delta \to 1$ case that we often consider.

In this environment, there are two main reasons why a person might not complete the project. The first revolves around whether the project is worth doing. The following definition will prove useful for describing such effects.

**Definition 3.** Given $\beta$ and $\delta$, stage 1 is $\beta$-worthwhile if $-c + \beta \delta \left( -k + \frac{\delta}{1-\delta} v \right) \geq 0$, and Stage 2 is $\beta$-worthwhile if $-k + \frac{\beta \delta}{1-\delta} v \geq 0$.

Stage $n \in \{1, 2\}$ is $\beta$-worthwhile if the person prefers completing the project starting from now as opposed to never completing the project. Given optimistic beliefs, stage 1 is $\beta$-worthwhile if the person prefers to complete stage 1 now and stage 2 next period as opposed to never starting the project. Stage 2 is $\beta$-worthwhile if a person who has completed stage 1 prefers to complete stage 2 now as opposed to never complete stage 2. Clearly a person will complete stage $n \in \{1, 2\}$ only if that stage is $\beta$-worthwhile. Moreover, because a person starts the project only if she believes that she will later finish it, and because given perceptions $\hat{\beta}$ she predicts she’ll complete stage 2 only if stage 2 is $\hat{\beta}$-worthwhile, the person will complete stage 1 only if stage 2 is $\hat{\beta}$-worthwhile. Notice that since TCs have time-consistent preferences, stage 1 being ($\beta = 1$)-worthwhile necessarily implies that stage 2 is ($\beta = 1$)-worthwhile. For a person with present-biased preferences, in contrast, it could be that stage 1 is $\beta$-worthwhile and yet stage 2 is not $\beta$-worthwhile.

Whether the project is worth doing in any of these senses is primarily driven by the person’s time-consistent impatience parameter $\delta$. In other words, it depends on whether the person gives sufficient weight to the future benefits to justify incurring the immediate costs. Indeed, for any $\beta$, $\hat{\beta}$, $c$, $k$, and $v$, there exists $\hat{\delta} < 1$ such that for all $\delta \geq \hat{\delta}$, stage 1 is $\beta$-worthwhile and stage 2 is both $\beta$-worthwhile and $\hat{\beta}$-worthwhile. In order to abstract away from such issues, we sometimes examine the limit case as $\delta$ approaches one, which we refer to as the “$\delta \to 1$ case”.

The second main reason why a person might not complete the project is “procrastination” — she views completion starting today as better than never completing the project, and she expects to complete the project, but she repeatedly plans to start completion in the near future rather than
now. Throughout this paper, we use this very precise definition of procrastination.12

**Definition 4.** A person procrastinates stage 1 if she never completes stage 1 despite it being $\beta$-worthwhile and stage 2 being $\hat{\beta}$-worthwhile; and when a person completes stage 1, she procrastinates stage 2 if she never completes stage 2 despite it being $\beta$-worthwhile.

The logic of procrastination in this environment is very much like that in O’Donoghue and Rabin (forthcoming). Given her self-control problem $\beta$, for each stage the person will have some maximum tolerable delay for completion of that stage. Formally, given $\beta$ and given optimistic beliefs, the maximum tolerable delay on stage $n \in \{1, 2\}$, which we denote by $d(\beta, n)$, is given by

$$d(\beta, 1) \equiv \max \left\{ d \in \{0, 1, \ldots\} \mid -c + \beta \delta \left( -k + \frac{\delta v}{1 - \delta} \right) < \beta \delta^{d+1} \left( -c - \delta k + \frac{\delta^2 v}{1 - \delta} \right) \right\}$$

$$d(\beta, 2) \equiv \max \left\{ d \in \{0, 1, \ldots\} \mid -k + \frac{\beta \delta v}{1 - \delta} < \beta \delta^{d+1} \left( -k + \frac{\delta v}{1 - \delta} \right) \right\}.$$ 

In words, $d(\beta, n)$ is the maximum delay $d$ such that the person prefers completing stage $n$ in $d$ periods rather than now.13

Whether the person chooses to delay depends on her predicted future behavior. Much as above, given her perception $\hat{\beta}$ of her future self-control problem, for each stage the person will perceive some maximum delay that she would tolerate in the future. With the notation above, given perceptions $\hat{\beta}$ and given optimistic beliefs, the perceived future tolerance for delay on stage $n$ is given by $d(\hat{\beta}, n)$. Because the maximum delay that a person could ever predict is $d(\beta, n)$ periods from next period, if $d(\hat{\beta}, n) < d(\beta, n)$ then the maximum delay she could ever predict for the future is tolerable, and therefore she never completes stage $n$. That is, the person procrastinates on stage $n \in \{1, 2\}$ if and only if her perceived future tolerance for delay of stage $n$, $d(\hat{\beta}, n)$, is at least one period shorter than her current tolerance for delay of stage $n$, $d(\beta, n)$. Because $\hat{\beta} \geq \beta$ implies $d(\hat{\beta}, 1) \leq d(\beta, 1)$, this means the person doesn’t procrastinate if and only if $d(\hat{\beta}, n) = d(\beta, n)$. The following lemma characterizes when this condition holds.

**Lemma 1.** $d(\hat{\beta}, 1) = d(\beta, 1)$ if $c \leq \frac{\beta \delta(1-\delta)}{1-\beta \delta} \left( -k + \frac{\delta v}{1 - \delta} \right)$ and only if $c \leq \frac{\beta \delta(1-\delta)}{1-\beta \delta} \beta \delta \left( -k + \frac{\delta v}{1 - \delta} \right)$. $d(\hat{\beta}, 2) = d(\beta, 2)$ if $k \leq \frac{\beta \delta(1-\delta)}{1-\beta \delta} \left( \frac{v}{1 - \delta} \right)$ and only if $k \leq \frac{\beta \delta(1-\delta)}{1-\beta \delta} \beta \delta \left( \frac{v}{1 - \delta} \right)$. 

Obviously, $d(\hat{\beta}, n) = d(\beta, n)$ for $\hat{\beta} = \beta$, and so TCs and sophisticates never procrastinate. For naifs, the condition for procrastination is quite simple. If the project is worth doing, then

12 This definition is equivalent to that in O’Donoghue and Rabin (forthcoming) and different from that in O’Donoghue and Rabin (1999a).

13 If $d(\beta, n)$ doesn’t exist, we write $d(\beta, n) = \infty$, which of course holds if and only if stage $n$ is not $\beta$-worthwhile.
that is, TCs would never delay. Because naifs perceive this to be their future behavior, they procrastinate stage $n$ whenever $d(\beta, n) > 0$ — that is, naifs procrastinate whenever they prefer completing stage $n$ next period to completing stage $n$ now. Lemma 1 implies that for naifs procrastination is monotonic in the stage cost. That is, for either stage, the larger is the cost, the more likely it is that naifs procrastinate.

For partial naifs, the conditions for procrastination are more complicated. The main reason is the discreteness of $d(\bar{\beta}, n)$ and $d(\beta, n)$. Partial naifs have a tendency to be more likely to procrastinate the higher is the stage cost. But there is a zone in which whether partial naifs procrastinate is non-monotonic in the stage cost. As we shall see, this non-monotonicity precludes strong conclusions for partial naifs.

We are now in a position to characterize perception-perfect strategies:

**Lemma 2.** For all $\delta, c, k,$ and $v$:

1. For $\beta \leq \bar{\beta} = 1$ (for TCs and naifs), there exists a unique perception perfect strategy; and

2. For $\beta \leq \hat{\beta} < 1$ (for sophisticates and partial naifs), if the person has optimistic beliefs, then either

   (a) There exists a unique perception-perfect strategy $s$, and $s$ satisfies $s(\emptyset, t) = 0$ for all $t$;

   (b) Any perception-perfect strategy $s$ satisfies $s(\emptyset, t) = 1$ for some $t \in \{1, 2, \ldots, d(\beta, 1) + 1\}$ but $s(t, \tau) = 0$ for all $\tau > t$; or

   (c) Any perception-perfect strategy $s$ satisfies $s(\emptyset, t) = 1$ for some $t \in \{1, 2, \ldots, d(\beta, 1) + 1\}$ and $s(t, t + 1) = 1$.

In our environment, there are three possible outcomes in terms of which stages the person completes: She can never start the project, she can complete stage 1 but not stage 2, and she can complete both stages 1 and 2. Lemma 2 establishes that we have uniqueness in which of these occurs. Part 1 establishes that there is a unique perception-perfect strategy for TCs and for naifs, and so clearly we have uniqueness in how many stages they complete. Part 2 establishes that, under the assumption of optimistic beliefs, we have also uniqueness for sophisticates and partial naifs in terms of how many stages they complete. More precisely, Lemma 2 establishes that multiple perception-perfect strategies arise only when these types complete stage 1, and the indeterminacy is solely in when they complete the first stage. In this paper, our focus is solely on how many stages people
Proposition 1 describes when a person completes the two stages:

**Proposition 1.** For all $\beta$, $\hat{\beta}$, $\delta$, $c$, $k$, and $v$, under any perception-perfect strategy with optimistic beliefs:

1. The person completes stage 1 if and only if stage 1 is $\beta$-worthwhile, stage 2 is $\hat{\beta}$-worthwhile, and $d(\hat{\beta}, 1) = d(\beta, 1)$; and

2. Conditional on completing stage 1, the person completes stage 2 if and only if stage 2 is $\beta$-worthwhile and $d(\hat{\beta}, 2) = d(\beta, 2)$.

Proposition 1 formalizes the points from our earlier discussion. There are three reasons a person might not start the project: She might feel the project is not worth doing (stage 1 is not $\beta$-worthwhile); she might predict that in the future she won’t find the project to be worth continuing (stage 2 is not $\hat{\beta}$-worthwhile); or she might procrastinate ($d(\hat{\beta}, 1) < d(\beta, 1)$). Similarly, conditional on starting the project, there are two reasons a person might not finish the project: She might feel the project is not worth continuing (stage 2 is not $\beta$-worthwhile); or she might procrastinate ($d(\hat{\beta}, 2) < d(\beta, 2)$).

While Proposition 1 characterizes observed behavior for any $\beta$ and $\hat{\beta}$, it is instructive to consider the implications of Proposition 1 for the extreme types often discussed in the literature: time-consistent people, people with present-biased preferences who are completely sophisticated, and people with present-biased preferences who are completely naive. We first consider TCs:

**Corollary 1.** Under their unique perception-perfect strategy, TCs — who have $\beta = \hat{\beta} = 1$ — either complete the project immediately or never start the project, and they complete the project immediately if and only if

$$-c - \delta k + \frac{\delta^2 v}{1 - \delta} \geq 0.$$ 

TCs never procrastinate, and moreover time-consistent preferences imply that if stage 1 is worth doing then so is stage 2. Hence, TCs either complete the project or never start the project, and which they do merely depends on whether the project is worth doing. The condition for whether TCs complete the project should look familiar; it is the standard net-present-value calculation applied to our special environment. In other words, TCs follow the standard rule of starting and later

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14 Moreover, as discussed in some detail in O’Donoghue and Rabin (forthcoming), there is a sense in which these delays in completing the first stage are not important....
completing a project if it yields positive net present value, and never starting a project if it yields negative net present value. An important implication is that, holding constant the stage-1 net present value — holding constant $-c - \delta k + \frac{\beta \delta^2 v}{1 - \delta}$ — changing the distribution of costs over the course of the project does not affect the behavior of TCs.

We next consider sophisticates:

**Corollary 2.** Under any perception-perfect strategy with optimistic beliefs, sophisticates — who have $\beta = \hat{\beta} < 1$ — either complete both stages or never start the project, and they complete both stages if and only if

$$-c - \beta \delta k + \frac{\beta \delta^2 v}{1 - \delta} \geq 0 \quad \text{and} \quad - k + \frac{\beta \delta v}{1 - \delta} \geq 0.$$

Sophisticates, like TCs, never procrastinate. But since sophisticates have time-inconsistent preferences, even when stage 1 is $\beta$-worthwhile, stage 2 might fail to be $\beta$-worthwhile. Of course, in such situations sophisticates never start the project because they correctly predict that they would never finish it. It follows that sophisticates either complete the project or never start the project, and which they do depends on whether both stages are $\beta$-worthwhile. Hence, whereas TCs follow the standard net-present-value rule, sophisticates follow a sort of modified net-present-value rule, modified in two ways. First, the net present value that matters is that which incorporates the person’s preference for immediate gratification — e.g., the first condition for sophisticates is identical to that for TCs except for the presence of $\beta$. Second, because of their time-inconsistent preferences, sophisticates must also confirm that stage 2 has a positive net present value, so that they expect to complete the project.

Finally, we consider naifs:

**Corollary 3.** Under their unique perception-perfect strategy, naifs — who have $\beta < \hat{\beta} = 1$ — either complete the project immediately, never start the project, or complete stage 1 immediately but never complete stage 2. They complete stage 1 if and only if

$$c \leq \frac{\beta \delta (1 - \delta)}{1 - \beta \delta} \left(-k + \frac{\delta v}{1 - \delta}\right),$$

and if they complete stage 1, then they complete stage 2 if and only if

$$k \leq \frac{\beta \delta (1 - \delta)}{1 - \beta \delta} \left(\frac{v}{1 - \delta}\right).$$

Because naifs perceive themselves to have time-consistent preferences, whenever stage 1 is $\beta$-worthwhile they expect to complete the project. Hence, unlike sophisticates, naifs would never not
start a $\beta$-worthwhile project because they fear not finishing it. But since $\hat{\beta} > \beta$, naifs might procrastinate on some stage. Because naifs don’t procrastinate a stage only if that stage is $\beta$-worthwhile, the condition which determines whether naifs complete a stage is that which determines whether they procrastinate that stage (from Lemma 1).

Analyzing the extreme assumptions of sophistication and naivete is useful because they illustrate the types of behavior that might arise for the more realistic case of partial naivete. Partial naifs exhibit elements of both sophistication (because $\hat{\beta} < 1$) and naivete (because $\hat{\beta} > \beta$). Hence, partial naifs, like sophisticates, might not start a project even when it is $\beta$-worthwhile because they (correctly) predict that they won’t finish it (since stage 2 is not $\hat{\beta}$-worthwhile). And partial naifs, like naifs, might procrastinate on either stage.

Two types of procrastination potentially arise when a person (who is not completely sophisticated) faces a long-term project. First, there is the classical form that we have seen in many papers wherein a person plans to complete a valuable project but delays starting it. But in addition, the person might start a valuable project planning to finish it, but delay finishing it. This latter form of procrastination is clearly worse than merely never initiating a valuable project, because the person incurs the cost associated with starting the project without ever accruing any benefits. In theory, there is in fact no bound on how much effort a person might exert in starting a project they do not finish, as reflected by the following result.

**Proposition 2.** For all $\beta, \gamma < 1$ and $\hat{\beta} > \beta$, for any $c$ there exists $k$ and $v$ such that the person completes stage 1 but never completes stage 2.

Interestingly, partial naifs can suffer worse outcomes than either extreme. Partial naifs are less likely to procrastinate any given stage than are naifs. But this means that in situations where naifs never start the project, if their partial sophistication overcomes procrastination of stage 1 but does not overcome procrastination of stage 2, then partial naifs start but do not finish the project. No matter whether sophisticates complete the project or never start in such situations, clearly both naifs and sophisticates experience better outcomes than do partial naifs.

As noted after Corollary 1, for TCs, the structure of costs over the course of the project is irrelevant: Holding constant the stage-1 net present value — holding constant $-c - \delta k + \frac{\delta^2 v}{1-\delta}$ — changing the distribution of costs over the course of the project does not affect the behavior of TCs. For people with present-biased preferences, in contrast, the cost structure is an important
determinant of their behavior. First, for a person who is (at least to some degree) sophisticated, the structure of costs can matter because it affects whether the person thinks she will complete stage 2 of the project. In particular, holding constant \( -c - \beta \delta k + \frac{\beta \delta^2 v}{1-\beta} \), decreasing \( c \) and increasing \( k \) makes it more likely that stage 2 is not \( \hat{\beta} \)-worthwhile, and therefore makes it less likely that the person completes stage 1.\(^{15}\)

The structure of costs matters more dramatically for whether a person procrastinates. In order to focus more directly on procrastination, we now consider the \( \delta \rightarrow 1 \) case. As discussed above, in this case everything is worthwhile, and therefore the only reason a person might not complete the project is procrastination.

**Lemma 3.** For the \( \delta \rightarrow 1 \) case:

1. \( d(\beta, 1) \equiv \max \{ d \in \{0, 1, \ldots\} \mid -c + \beta (dv) < -\beta c \} \) and
   \[ d(\beta, 2) \equiv \max \{ d \in \{0, 1, \ldots\} \mid -k + \beta (dv) < -\beta k \} \]
2. \( d(\hat{\beta}, 1) = d(\beta, 1) \) if \( c \leq \frac{\beta v}{1-\beta} \) and only if \( c \leq \frac{\beta v}{1-\beta/\hat{\beta}} \), and \( d(\hat{\beta}, 2) = d(\beta, 2) \) if \( k \leq \frac{\beta v}{1-\beta} \) and only if \( k \leq \frac{\beta v}{1-\beta/\hat{\beta}} \).

Lemma 3 establishes some convenient features of the \( \delta \rightarrow 1 \) case. Unlike for the \( \delta < 1 \) case, the maximum tolerable delay for stage 1 is independent of the cost of stage 2. As a result, for either stage, whether the person procrastinates that stage is independent of the cost of the other stage, and is also independent of the order of the stages. In addition, just as before, whether naifs procrastinate a stage is monotonic in the stage costs — the bigger is the stage cost, the more likely it is that naifs procrastinate that stage. This tendency also holds for partial naifs, but once again there is a zone of non-monotonicities that precludes general results.\(^{16}\)

Proposition 3 uses Lemma 3 to characterize how the different types behave in the \( \delta \rightarrow 1 \) case:

\(^{15}\) We doubt, however, the importance of such effects; for instance, if such a reallocation leads the person not to complete the project for these reasons, the degree of harm can be large only if \( \beta \) is significantly less than 1.

\(^{16}\) For instance, suppose \( \beta = .9, \hat{\beta} = .91, v = 1, x = 10, \) and \( y = 15 \). If stage \( x \) has cost \( x \), then \( d(\beta, x) = 0 < 1 = d(\hat{\beta}, x) \), and therefore the person procrastinates stage \( x \). If stage \( y \) has cost \( y \), then \( d(\beta, y) = 1 = d(\hat{\beta}, y) \), and therefore the person does not procrastinate stage \( y \). Hence, even though \( y > x \), partial naifs with \( \beta = .9 \) and \( \hat{\beta} = .91 \) would procrastinate stage \( x \) but not stage \( y \).
Proposition 3. For the \( \delta \to 1 \) case, for any \( c, k, \) and \( v \):

1. If \( \beta = \hat{\beta} \leq 1 \) (for TCs and sophisticates), the person completes the project;

2. If \( \beta < \hat{\beta} = 1 \) (for naifs), the person completes stage 1 if and only if \( c \leq \frac{\beta v}{1-\beta} \), and if she completes stage 1, then she completes stage 2 if and only if \( k \leq \frac{\beta v}{1-\beta} \), and

3. If \( \beta < \hat{\beta} < 1 \) (for partial naifs), the person completes stage 1 if \( c \leq \frac{\beta v}{1-\beta} \) and only if \( c \leq \frac{\beta v}{1-\beta/\beta} \), and if she completes stage 1, then she completes stage 2 if \( k \leq \frac{\beta v}{1-\beta} \) and only if \( k \leq \frac{\beta v}{1-\beta/\beta} \).

Proposition 3 part 1 reflects that for the \( \delta \to 1 \) case, everything is worth doing, and so TCs and sophisticates both complete the project (although sophisticates may have a short delay). Parts 2 and 3 characterize the behavior of naifs and partial naifs. Since everything is worth doing, the only reason naifs and partial naifs might not complete the project is if they procrastinate. A straightforward application of Lemma 3 generates the conditions for the two types.

Proposition 3 has important implications for how the cost structure affects procrastination. For simplicity, consider naifs, although similar intuitions apply for partial naifs. Because Proposition 3 implies that naifs complete the project if and only if \( \max\{c, k\} \leq \frac{\beta v}{1-\beta/\beta} \), the allocation of costs over the course of the project becomes crucial. In particular, for any fixed total cost, naifs are most likely to complete the project when the costs are allocated evenly over the course of the project — i.e., if total costs are \( \Gamma \), naifs are most likely to complete the project when \( c = k = \Gamma/2 \). As costs are allocated more unevenly, naifs are less likely to complete the task — e.g., naifs are least likely to complete the project when \( c = \Gamma \) or \( k = \Gamma \). The intuition for the importance of allocation is simple: Naifs complete the project if and only if they don’t procrastinate the highest-cost stage. If a disproportionate share of the costs are allocated to stage 1, then the person is prone to procrastinate starting the project; and if a disproportionate share of the costs are allocated to stage 2, then the person is prone to procrastinate finishing the project.

When costs are allocated unevenly, the order of costs becomes important, because it determines whether naifs incur costs without accruing benefits. If both costs are sufficiently low that the person wouldn’t procrastinate either stage, or if both costs are sufficiently high that the person would procrastinate both stages, then the order of costs is irrelevant. But if costs are such that the person would procrastinate the high-cost stage but not the low-cost stage, then the person never starts when the high-cost stage comes first, whereas the person starts but doesn’t finish when the high-cost stage comes second. Hence, naifs are better off when the high-cost stage comes first —
that is, when \( \min\{c, k\} = k \).

Similar intuitions hold for partial naifs. Because partial naifs, like naifs, are more prone to procrastinate a stage the higher is the cost of that stage, the allocation of costs and order of costs can matter for partial naifs in the same way that it matters for naifs. But since partial naifs have the non-monotonic region, it is possible to construct examples where the conclusions go the other way.

4. Model with Endogenous Cost Structure

The previous section shows how the behavior of people with present-biased preferences depends critically on the structure of costs over the course of a project. In this section, we endogenize this structure. We consider two cases. First, we suppose the costs of each stage are fixed, as in the basic model, but we allow the person to choose the order in which she completes the stages. Second, we allow the person to choose the allocation of costs over the course of the project — e.g., an even distribution vs. an uneven distribution. We show that in such situations people are prone to choose cost structures for which they are likely to start but not finish the project.

In order to endogenize the order of stages, we suppose that, as in our basic model, a long-term project consists of two stages, but that, unlike in our basic model, the person can complete these stages in either order. Let the immediate costs for the two stages be \( x \) and \( y \) with \( y \geq x > 0 \). We often refer to the stages as “stage \( x \)” and “stage \( y \)”. As before, upon completion of both stages the person receives an infinite stream of benefits with per-period benefit \( v \geq 0 \). Importantly, the choice of what order to complete the stages is made at the moment of completion. Hence, in period 1, for instance, the person has three options: do nothing, complete stage \( x \) (planning to complete stage \( y \) in the future), or complete stage \( y \) (planning to complete stage \( x \) in the future).

In this environment, the person is effectively making a choice between two possible projects, a project which involves first completing stage \( x \) and then completing stage \( y \), which we denote by \((x, y)\), and a project which involves first completing stage \( y \) and then completing stage \( x \), which we denote by \((y, x)\). For situations like this in which a person must choose between multiple projects, we let \( P \) denote the set of projects available. For our endogenous-order environment, \( P = \{(x, y), (y, x)\} \). Given a set of projects \( P \), we let \( p^*(P) \) be the person’s optimal project. Formally, \( p^*(P) \equiv \arg \max_{(c,k) \in P'(P)} \left[-c - \beta \delta k + \frac{\beta}{1 - \delta} v \right] \) where \( P'(P) \equiv \{p \in P | \text{stage 2 is } \beta\text{-worthwhile}\} \). In words, the person’s optimal project is the project that maximizes her preferences.
subject to the condition that she expects to complete it (the latter condition matters only for \( \hat{\beta} < 1 \)). Clearly, if a person starts any project, it will be her optimal project. The following lemma characterizes a person’s optimal project.

**Lemma 4.** For any \( \beta, \hat{\beta}, \delta, v, \) and \( x \leq y, \) if \( P = \{(x, y), (y, x)\} \) then \( p^*(P) = (x, y). \)

Hence, the optimal project for all types involves completing the low-cost stage first and the high-cost stage second. There are two forces behind this conclusion, both of which militate in favor of completing the low-cost stage first. First, the person’s time-consistent impatience, as captured by \( \delta, \) leads her to prefer incurring costs in the future rather than now, and hence, conditional on total costs being the same, she prefers to complete the low-cost stage first. Second, even if there were no time-consistent impatience — for \( \delta \) close to one — the person’s preference for immediate gratification, as captured by \( \beta, \) also leads her to prefer incurring costs in the future rather than now.

Lemma 4 in conjunction with Corollaries 1 and 2 imply that TCs and sophisticates either complete project \((x, y)\) or never start anything. Lemma 4 in conjunction with Corollary 3 implies that naifs and partial naifs might complete project \((x, y)\), never start anything, or complete stage \(x\) but then never complete stage \(y\). In order to explore the implications of endogenizing the order of completion relative to an exogenous cost structure, we compare a person’s behavior given exogenous order — i.e., given \( P^{\text{exog}} \in \{(x, y), (y, x)\}\) — to her behavior given endogenous order — i.e., given \( P^{\text{endog}} = \{(x, y), (y, x)\}\). The following proposition makes such a comparison for TCs and sophisticates.

**Proposition 4.** Let \( P^{\text{exog}} \in \{(x, y), (y, x)\}\) and \( P^{\text{endog}} = \{(x, y), (y, x)\}\). For any \( \beta, \delta, v, \) and \( x \leq y, \) for \( \hat{\beta} = \beta \leq 1 \) (for TCs and sophisticates), if the person completes both stages given \( P^{\text{exog}} \), then she completes both stages given \( P^{\text{endog}}. \)

Proposition 4 establishes that TCs and sophisticates are more likely to complete both stages when the order is endogenous. The intuition is that for any fixed stage costs \(x\) and \(y\), having the low-cost stage first makes it more likely that the project is worth doing. This intuition holds for naifs and partial naifs as well, and so for \( \delta < 1 \) they may be more likely to complete both stages when the order is endogenous. But endogenizing the order might also influence whether naifs and partial naifs procrastinate (as before, TCs and sophisticates never procrastinate). In order to focus

\[17\] In all examples that we consider, the person’s optimal project is also what she plans to complete in the future in the event of current delay. This need not be true more generally.
directly on the implications of endogenous order for procrastination, we consider the $\delta \to 1$ case, in which case the above intuition disappears. The following proposition describes the implications for procrastination of endogenizing the order of completion.

**Proposition 5.** Let $P_{\text{exog}} \in \{(x, y), (y, x)\}$ and $P_{\text{endog}} = \{(x, y), (y, x)\}$, and consider the $\delta \to 1$ case. For any $\beta, v, x \leq y$:

1. If $\hat{\beta} = \beta \leq 1$ (for TCs and sophisticates), the person completes both stages for both $P_{\text{exog}}$ and $P_{\text{endog}}$;

2. If $\beta < \hat{\beta} = 1$ (for naifs), the person completes both stages with $P_{\text{endog}}$ if and only if she completes both stages with $P_{\text{exog}}$, but when she doesn’t complete both stages, she completes one stage with $P_{\text{endog}}$ if she completes one stage with $P_{\text{exog}}$.

Proposition 5 part 1 merely reiterates our earlier point that for the $\delta \to 1$ case, everything is worth doing, and so TCs and sophisticates complete the project. Proposition 5 part 2 establishes that for naifs, endogenizing the order of completion does not change the likelihood of the person actually completing the project, but when the person does not complete the project, endogenizing the order of completion makes it more likely that she starts the project — that she exhibits our new form of procrastination wherein she incurs costs without ever receiving benefits. The intuition is simple: In Section 3, we saw for the case of an exogenous cost structure that naifs incur costs without benefits only if a project involves initial low costs and later high costs, and with endogenous order this is exactly the order that naifs choose.

For partial naifs, a similar intuition can hold. That is, just like naifs, endogenizing the order of completion makes partial naifs choose to complete the low-cost stage first and the high-cost stage second, and hence can lead them to incur costs without benefits when they otherwise would not have started the project. But because of the non-monotonicities discussed in the previous section, the alternative conclusion can also hold for partial naifs — one can construct examples in which partial naifs procrastinate stage $y$ but not stage $x$ despite stage $y$ having a larger cost, and therefore endogenizing the order can lead partial naifs to never start when they otherwise would have incurred costs without benefits.

While our analysis above endogenizes the order of the stages for fixed stage costs, we next modify our model to endogenize the allocation of costs over the course of the project. In light of our conclusions above, we focus on the case where the project involves an initial low-cost stage and a later high-cost stage. But we now introduce choice over the allocation of costs. For simplicity
— and in part to reflect constraints on possible allocations — we assume the person must choose between two cost allocations, \((x_1, y_1)\) and \((x_2, y_2)\). We assume that the projects have the same total cost but that the first project has a more unequal allocation, so \(x_1 + y_1 = x_2 + y_2\) but \(x_1 < x_2 \leq \frac{1}{2} \leq y_2 < y_1\). Upon completion of both stages of either project, the person receives an infinite stream of benefits with per-period benefit \(v \geq 0\). As above, the allocation choice is made at the moment of completion. Hence, in period 1, for instance, the person has three options: do nothing, complete stage \(x_1\) (planning to complete stage \(y_1\) in the future), or complete stage \(x_2\) (planning to complete stage \(y_2\) in the future).

In this endogenous-allocation environment, the person is again effectively making a choice between two possible projects, where here \(P = \{(x_1, y_1), (x_2, y_2)\}\). The following lemma characterizes a person’s optimal project in this environment.

**Lemma 5.** For any \(\beta, \hat{\beta}, \delta, v, x_1 < x_2 \leq \frac{1}{2}\), and \(y_1\) and \(y_2\) such that \(x_1 + y_1 = x_2 + y_2\), if \(P = \{(x_1, y_1), (x_2, y_2)\}\) then:

1. If \(\hat{\beta} = 1\) (for TCs and naifs), then \(p^*(P) = (x_1, y_1)\); and
2. If \(\hat{\beta} < 1\) (for sophisticates and partial naifs), then \(p^*(P) = (x_1, y_1)\) unless stage \(y_1\) is not \(\hat{\beta}\)-worthwhile, stage \(x_2\) is \(\hat{\beta}\)-worthwhile, and stage \(y_2\) is \(\hat{\beta}\)-worthwhile.

Lemma 5 reflects that, when the allocation of costs is endogenous, people have a tendency to choose an uneven allocation. The intuition behind this result is the same as that behind Lemma 4: Both time-consistent impatience, as captured by \(\delta\), and a preference for immediate gratification, as captured by \(\beta\), lead a person to prefer incurring costs in the future rather than now, and hence, conditional on total costs being the same, she prefers to allocate as much as possible to the second stage. There is a caveat, however, to this basic intuition: If a person expects to have future self-control problems — if \(\hat{\beta} < 1\) — then she will not allocate so much cost to stage 2 so as to make her future self not find stage 2 to be worth doing. In other words, a person will choose the uneven allocation only if she expects to complete both stages given that allocation.

Proposition 6 examines the implications of endogenizing the allocation of costs (relative to an exogenous cost structure) for TCs and sophisticates.
Proposition 6. Let $\mathbf{P}^{\text{exog}} \in \{(x_1, y_1), (x_2, y_2)\}$ and $\mathbf{P}^{\text{endog}} = \{(x_1, y_1), (x_2, y_2)\}$. For any $\beta$, $\delta$, $v$, $x_1 < x_2 \leq \frac{1}{2}$, and $y_1$ and $y_2$ such that $x_1 + y_1 = x_2 + y_2$, for $\hat{\beta} = \beta \leq 1$ (for TCs and sophisticates), if the person completes both stages given $\mathbf{P}^{\text{exog}}$, then she completes both stages given $\mathbf{P}^{\text{endog}}$.

Proposition 6 establishes that, as when order is endogenous, TCs and sophisticates are more likely to complete both stages when the allocation of costs is endogenous. The intuition is that for a fixed total cost, allocating more of that cost to the second stage makes it more likely that the project is worth doing. Once again, this intuition holds for naifs and partial naifs as well, and so for $\delta < 1$ they may be more likely to complete both stages when the allocation of costs is endogenous. In order to focus directly on the implications of endogenous allocation for procrastination, we consider the $\delta \to 1$ case, in which case the above intuition disappears. The following proposition describes the implications for procrastination of endogenizing the allocation of costs.

Proposition 7. Let $\mathbf{P}^{\text{exog}} \in \{(x_1, y_1), (x_2, y_2)\}$ and $\mathbf{P}^{\text{endog}} = \{(x_1, y_1), (x_2, y_2)\}$, and consider the $\delta \to 1$ case. For any $\beta$, $v$, $x_1 < x_2 \leq \frac{1}{2}$, and $y_1$ and $y_2$ such that $x_1 + y_1 = x_2 + y_2$:

1. If $\hat{\beta} = \beta \leq 1$ (for TCs and sophisticates), the person completes both stages for both $\mathbf{P}^{\text{exog}}$ and $\mathbf{P}^{\text{endog}}$;

2. If $\hat{\beta} = 1$ (for naifs), the person completes both stages with $\mathbf{P}^{\text{endog}}$ only if she completes both stages with $\mathbf{P}^{\text{exog}}$, and she completes at least one stage with $\mathbf{P}^{\text{endog}}$ if she completes at least one stage with $\mathbf{P}^{\text{exog}}$.

Proposition 7 part 1 again reiterates that TCs and sophisticates always complete the project in the $\delta \to 1$ case. Proposition 7 part 2 establishes that for naifs, endogenizing the allocation of costs makes it more likely that they will start the project, while at the same time they are less likely to complete the project. The intuition is again quite simple: In Section 3, we saw for the case of an exogenous cost structure that naifs are most likely to complete the project for an even allocation of costs over the course of the project, but with endogenous allocation naifs choose an uneven allocation.

For partial naifs, a similar intuition can hold. That is, just like naifs they choose an uneven allocation of costs, and hence are prone to start but not finish the project. But once again the non-monotonicities permit endogenizing the order to have essentially any effect — one can construct examples in which partial naifs exhibit any permutations of never start, start but don’t finish, and complete both stages when the allocation of costs is endogenized.
A natural extension of our endogenous allocation model is to allow the allocation chosen to affect total costs. In particular, if the choice is between high effort for a short duration vs. low effort for a long duration, it seems quite plausible that total costs differ. Of course, it is not a priori clear which is more efficient. If there are “start-up costs” — e.g., when the person sits down to work, she must review what project is about, what she has done already, etc. — then it would be more efficient to put in high effort for a short duration. If, in contrast, there are “decreasing returns to effort” — e.g., the eighth hour of work on a given day is less effective than the fourth hour — then it would be more efficient to put in low effort for a longer duration. To examine such effects, suppose a person is choosing between a one-period project with cost $x_1$ vs. a two-period project with costs $x_2$ and $y_2$. We assume $x_2 \leq y_2 < x_1$, reflecting that project 1 is the high-effort option. But we allow for both $x_1 < x_2 + y_2$ and $x_1 - v > x_2 + y_2$; these two cases reflect that one project is unambiguously more efficient than the other. The following proposition characterizes behavior for naifs:

**Proposition 8.** Let $\mathbf{P}^{\text{exog}} \in \{(x_1)\}, \{(x_2, y_2)\}$ and $\mathbf{P}^{\text{endog}} = \{(x_1), (x_2, y_2)\}$, and consider the $\beta \rightarrow 1$ case. For $\beta < \hat{\beta} = 1$ (for naifs), for any $v$ and $x_2 \leq y_2 < x_1$:

1. If $x_1 < x_2 + y_2$, then $\mathbf{p}^*(\mathbf{P}^{\text{endog}}) = (x_1)$, and the person completes the project with $\mathbf{P}^{\text{endog}}$ only if she completes the project with $\mathbf{P}^{\text{exog}}$; and

2. If $x_1 - v > x_2 + y_2$, then $\mathbf{p}^*(\mathbf{P}^{\text{endog}}) = (x_2, y_2)$, and the person completes the project with $\mathbf{P}^{\text{endog}}$ if she completes the project with $\mathbf{P}^{\text{exog}}$.

Proposition 8 establishes how the interaction of cost allocation and efficiency matters for procrastination. In general, naifs plan to complete the more efficient project (slight caveat needed here). If it is more efficient to have an unequal allocation of costs, then there is a second force that leads naifs to choose the uneven allocation, and hence further reinforces our conclusion that endogenizing the allocation of costs makes it less likely naifs complete the project. If, in contrast, it is more efficient to have an equal allocation, then there is a force which counteracts the tendency to choose an uneven allocation, and hence mitigates our conclusion that endogenizing the allocation of costs makes it less likely naifs complete the project (although in this case it’s possible that choosing low effort for a longer duration leads them to start but not finish when they would have otherwise not started).

Our results in this section can be interpreted in terms of the intuition identified in O’Donoghue
and Rabin (forthcoming). The basic intuition in that paper is that when, in addition to deciding when to do something, a person must also make a choice over what to do, the person can be more prone to procrastinate. In this section, we have analyzed a variety of ways in which a person might face choice over which long-term project to do, and we have seen how this choice can make the person more prone to procrastinate. But there exists a sense in which the results here are more compelling. Our point here is that when we endogenize the cost structure, people are prone to choose cost structures on which they are more prone to procrastinate — and in fact the more costly form of procrastination where incur some of the costs without ever incurring any benefits....

5. Ongoing Procrastination Costs

In this section we consider two extensions of our model. First, we suppose that a person has multiple long-term projects to complete. Second, we suppose that if the person chooses to delay, she must pay a small cost to keep open the option of completing the project in the future. Although we discuss results for other types, for simplicity we confine our formal analysis to naifs.

As before, suppose that a long-term project consists of two stages. But now suppose that a person faces \( N \) long-term projects that she might complete. For simplicity, we assume that all projects are identical: Each project has cost structure \( (c, k) \), and completion of a project in period \( \tau \) initiates a stream of benefits \( v > 0 \) in each period from \( \tau + 1 \) onward. Importantly, in each period the person can complete at most one stage, and hence she has three options: start a new project (if not all projects have been started), complete a started project (if one exists), and do nothing.

To focus on the implications of there being multiple projects for procrastination, we examine the \( \delta \rightarrow 1 \) case. In this environment, it is straightforward to show that TCs start and finish all \( N \) projects in succession — that is, in period 1 they start project 1, in period 2 they finish project 1, in period 3 they start project 2, in period 4 they finish project 2, and so forth until they have completed all \( N \) projects. Intuitively, for the \( \delta \rightarrow 1 \) case all projects are worth doing, and moreover it always better to finish a started project before starting a new project so as to initiate the benefits of each project as soon as possible.

Naifs, of course, believe that in the future they will behave according to this intuition, but their preference for immediate gratification might lead them to do something different now. Lemma 6
characterizes a naive person’s preferences over her three possible current actions, doing nothing, starting a new project, and finishing an already started project.

**Lemma 6.** Consider the $\delta \to 1$ case, and suppose $\beta < \hat{\beta} = 1$. If in period $t$ there are $n$ projects that the person has not started and $m$ projects that the person has started but not finished, then the person’s preferences over her three (possibly) available actions are as follows:

- Start new project $\succeq$ do nothing if and only if $c \leq \frac{1}{1-\beta}nv$
- Finish started project $\succeq$ do nothing if and only if $k \leq \frac{1}{1-\beta}(n + m)v$
- Finish started project $\succeq$ start new project if and only if $k - c \leq \frac{1}{1-\beta}mv$

The first two conditions in Lemma 6 are very much like our conditions for procrastination of the two stages in the one-project model. When there is only one project, naifs believe for either stage that doing nothing now delays the eventual completion of the project by one period, and hence reduces the benefits by $v$. With multiple projects, a similar intuition holds, but it is a little richer.

First consider a naive person’s preference between starting a new project vs. doing nothing. If a naive person has $n$ unstarted projects, she believes that, relative to starting a new project, doing nothing rather than starting a new project delays the eventual completion of each of the $n$ unstarted projects by one period, and hence reduces total benefits by $nv$. This conclusion holds even if she also has $m > 0$ started but not yet finished projects: In that case, no matter whether she starts a new project or does nothing now, she plans to complete the $m$ previously started projects beginning next period, and so $m$ is irrelevant to her preference between starting a new project vs. doing nothing.

Next consider her preference between finishing a started project vs. doing nothing. If a naive person has $m$ started but not finished projects and $n$ unstarted projects, then she believes that doing nothing rather than finishing a started project delays the eventual completion of each of the $m + n$ unfinished projects by one period, and hence reduces total benefits by $(m + n)v$. The first two conditions in Lemma 6 imply that having multiple projects to complete can help motivate a person not to procrastinate, because delay of the current project imposes a delay on all projects she expects to complete in the future. We discuss this intuition elsewhere (e.g., O’Donoghue and Rabin (forthcoming)). But while this force helps counteract procrastination of the form we’ve discussed, it certainly doesn’t eliminate it. For instance, one can construct examples in which the person starts all $N$ projects and doesn’t finish any of them (as will become clear below).

The third condition in Lemma 6 reflects that a new distortion can arise in this environment. Whereas the person should, as discussed above, finish started projects before she starts new projects,
if $k$ is large enough relative to $c$, she might instead start new projects before she finishes started projects. In other words, the person might delay completion of started projects not by doing nothing, but rather by inefficiently starting a new project. This new distortion is driven by the same preference for immediate gratification that generates procrastination: In environments in which a person must choose which of a variety of ongoing projects to work on, a preference for immediate gratification distorts her decision away from high-cost projects and towards low-cost projects. In the environment here, this distortion can lead a person to carry around an inventory of started but not completed projects.

Proposition 9 uses Lemma 6 to characterize the behavior of naifs in this environment:

**Proposition 9.** Consider the $\delta \to 1$ case, and suppose $\beta < \hat{\beta} = 1$. For any $c < k$ and $v$, there exists $n_S, n_F \leq n_S$, and $n_I \in \{n_S - n_F - 1, n_S - n_F\}$ such that naifs start $n_S$ projects, finish $n_F$ projects, and carry an inventory of $n_I$ projects.

The following examples illustrate this behavior:

If $n_S = 3$, $n_F = 2$, and $n_I = 1$, then behavior path is
(start, start, finish, start, finish, $\emptyset$, $\emptyset$, ...).

If $n_S = 4$, $n_F = 2$, and $n_I = 1$, then behavior path is
(start, start, finish, start, finish, start, $\emptyset$, $\emptyset$, ...).

If $n_S = 5$, $n_F = 2$, and $n_I = 3$, then behavior path is
(start, start, start, start, finish, start, finish, $\emptyset$, $\emptyset$, ...).

Before leaving this extension, we briefly discuss the behavior of sophisticates in the multiple-project environment. Because for the $\delta \to 1$ case all projects are worth doing, sophisticates eventually complete all $N$ projects. As in the basic model, sophisticates might have short delays on some or all projects. More interesting, sophisticates might exhibit the distortions of the type discussed above for naifs wherein they start new projects before finishing old projects, and therefore carry around an inventory of started but unfinished projects (although unlike naifs sophisticates eventually complete all projects). E.g., if $n_I = 3$ (where $n_I$ is that for naifs), then under the perception-perfect strategy with no delays (which always exists), sophisticates follow behavior path (start, start, start, start, finish, start, finish, ...; start, finish, finish, finish, finish).
Our second extension returns to the realm of one project, but we now suppose that if the person chooses to delay, she must pay a small cost to keep open the option of completing the project in the future. Suppose again that a long-term project consists of two stages, where the cost structure is \((c, k)\) and completion of the project in period \(\tau\) initiates a stream of benefits \(v > 0\) in each period from \(\tau + 1\) onward. But now suppose that if the person wants to do nothing while at the same time she wants to keep open the option of completing the project in the future, she must pay a small fee \(\pi > 0\). Hence, in any period before which the person has neither started the project nor jettisoned the project, she has three options: she can start the project at cost \(c\), she can do nothing but keep the project open at cost \(\pi\), or she can jettison the project. Similarly, in any period before which the person started the project and kept it open, she has three options: she can finish the project at cost \(k\), she can do nothing but keep the project open at cost \(\pi\), or she can jettison the project.18

As for our first extension, we focus on the implications of such an option cost for procrastination, and hence we examine the \(\delta \rightarrow 1\) case. Without the option cost, TCs and sophisticates complete the project, and clearly the option cost merely makes them more motivated to complete the project (although sophisticates might exhibit a short delay and hence pay some option costs). The following proposition describes the behavior of naifs:

**Proposition 10.** Consider the \(\delta \rightarrow 1\) case, and suppose \(\beta < \tilde{\beta} = 1\). The person starts the project if and only if \(c \leq \frac{\beta v + \pi}{1 - \beta}\), and if she starts the project then she finishes the project if and only if \(k \leq \frac{\beta v + \pi}{\beta + \pi}\). Moreover, the person never jettisons the project — and so if she doesn’t start the project she pays the option cost \(\pi\) in all periods and if she starts but doesn’t finish the project she incurs the stage-1 cost \(c\) in period 1 and then pays the option cost \(\pi\) in all subsequent periods.

A comparison of Proposition 10 to Corollary 3 reveals that the option cost makes the person more motivated to complete the project. The intuition is obvious: Given \(\delta \rightarrow 1\), the project is well worth doing, and so the person never even considers jettisoning the project; and since doing nothing is now costly, the person is less prone to do nothing. But if this extra motivation is not enough to prevent procrastination, the person suffers a particularly unfortunate outcome: She pays the option cost \(\pi\) in (almost) all periods, because she persistently plans to complete the project in the near future, when in fact she never gets around to completing the project and reaping the anticipated benefits.

18 While our analysis treats \(\pi\) as an immediate cost, if \(\pi\) were a monetary cost then it would be better to treat \(\pi\) as a future cost (forgone future consumption). Our qualitative results are independent of which modeling choice is used, as the interpretation only affects the magnitude of the option cost that the person is willing to pay.
Our conclusion that people might pay significant costs to keep the option to carry out beneficial projects is not unique to long-term projects. Even for one-stage projects a naive person might repeatedly pay an option cost without ever carrying out the project. But our analysis in Section 4 suggests people may be particularly prone to this problem in realm of long-term projects — in particular, our conclusions that people are prone to choose cost structures that make it likely that they’ll start projects but not finish them. Moreover, if we extrapolate from our model and incorporate another behavioral phenomenon, the fact that people are prone to incur the costs of starting a project before they procrastinate, combined with the tendency to care about sunk costs, may make people even more prone to pay option costs to keep partially completed projects open.

6. Discussion and Conclusion

7. Appendix A: Analysis of Other Beliefs

8. Appendix B: Proofs
References


