Optimal Income Taxation of Couples

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Abstract

This paper analyzes optimal joint income taxation of two-earner couples. Each couple is modelled as a single rational economic agent supplying labor along two dimensions: primary and secondary earnings. In contrast to the previous literature, we consider fully general joint income tax systems. The optimum tax system is separable into two individual and independent tax schedules only if skills, labor supply, and social marginal welfare weights are also separable between the two members of each couple. Strikingly, for a wide class of social welfare objectives, the tax distortion on the secondary earner vanishes asymptotically when the earnings of the primary earner become large. Numerical simulations (in progress) show that the tax rate on the secondary earner is actually decreasing with the earnings of the primary earner. As a result, the marginal tax rate on the primary earner is actually lower when the secondary earner works. Such a schedule is optimal because it allows the government to redistribute more from two earner toward one earner couples when the primary earner income is lower.
1 Introduction

The treatment of couples for income tax purposes has attracted substantial attention among tax policy makers. Over the last three decades, a number of European countries have switched from joint taxation, whereby income taxes are assessed at the family level and hence depend on total income in the family, toward individual taxation, where income taxes are assessed at the individual level and hence are independent of the earnings of other members of the family. Two simple points have been noted in the previous informal discussions among economists of the problem (see e.g., Pechman, 1987 and Rosen, 1977).

First, the empirical labor supply literature has shown that the labor supply of secondary earners is more elastic with respect to tax rate than the labor supply of primary earners (see e.g., Blundell and MaCurdy, 1999 for a recent survey). Therefore, following the traditional Ramsey optimal taxation principle, labor income of secondary earners should be taxed relatively less than labor income of primary earners for efficiency reasons. This goal is achieved to some extent by a progressive income tax based on individuals as primary earners have higher incomes and hence will face higher marginal tax rates than secondary earners. In contrast, a family based income tax generates identical marginal tax rates for all members of the same family and thus does not meet this efficiency principle.

Second, welfare is better measured by family income rather than individual income. As a result, if the government values redistribution, two married women with the same labor income ought not to be treated identically if their husbands’ incomes are very different. This redistributive principle is achieved to some extent by progressive income taxation based on family income which imposes a higher tax rate on wives married to high income husbands than lower income husbands. In contrast, an individually based income tax imposes the same tax burden on wives, irrespective of their husbands’ earnings and hence does not meet this redistributive criterion.1

The purpose of this paper is to analyze the problem of the taxation of couples from an optimal income tax perspective. The optimal income tax theory following the seminal contribution of Mirrlees (1971) has focused almost exclusively on individuals rather than families. There is

1 Another important topic that is often discussed is the neutrality of the tax system with respect to marriage decisions. This paper considers only couples and hence does not tackle the neutrality issue.
little previous work on the optimal income taxation of families. The most important previous contribution is Boskin and Sheshinski (1983). They consider linear taxation of couples with possibly differentiated tax rates. Thus, their problem is formally identical to a many-person Ramsey optimal tax problem. They analyze formally the efficiency principle discussed above and provide a number of useful numerical simulations using empirical labor supply elasticities. Their results show that the tax rate on secondary earners should indeed be much lower than on primary earners. However, because they restrict themselves to linear taxation, their tax system is fundamentally an individual (albeit gender specific) income tax. Thus, they cannot address the central question of how the tax rate on one earner should depend on the earnings of his/her spouse.

In this paper, we consider fully general income tax systems which can depend on the earnings of each spouse in any nonlinear way and hence impose no a-priori restrictions. Such a problem can be seen as a multi-dimensional screening problem where agents (couples in the present paper) are characterized by a multi-dimensional parameter (the ability and tastes for work parameters of each member of the couple) that are unobserved by the Principal (the government maximizing social welfare).

There are very few studies in the optimal taxation literature tackling multi-dimensional screening problems. Mirrlees (1976, 1986) considered briefly such general screening problems in the context of optimal taxation but did not go beyond obtaining general first order conditions and did not consider specifically the case of family taxation. More recently, Cremer, Pestieau, and Rochet (2001), revisit the issue of commodity versus income taxation in a multi-dimensional screening model in a finite type economy. The nonlinear pricing literature in the industrial organization literature (see Wilson, 1993 for an extensive exposition) has investigated a number of aspects of multi-dimensional screening problems. Wilson (1993), Armstrong and Rochet (1999), and Rochet and Stole (2003) provide surveys of this literature. Multi-dimensional screening problems are difficult to analyze because, in contrast to the one-dimensional case, first order conditions are not sufficient to characterize the optimal solution in general. In this paper, we consider models with a discrete number of earnings outcomes for the secondary earner which simplifies the theoretical analysis and allows us to characterize optimal solutions using the standard first order approach. Furthermore, we are able to derive a number of asymptotic properties of optimal schedules which are relevant for tax policy analysis.
and that, to the best of our knowledge, have not been analyzed in the nonlinear pricing theory.

In the paper, we consider a simple model of family labor supply. In order to simplify the analysis, as in the nonlinear pricing literature, we assume away income effects and assume separability in the disutility of supplying labor between the two members of the couple. We first show that, if the abilities of each member of the couple are distributed independently, then the optimal tax schedule is separable into two individualistic tax schedules if and only if the social marginal welfare weights for each couple are also separable between the two members of the couple. For standard redistributive social welfare functions, this separability assumption will not hold, as a wife with a high income husband will typically be given a lower marginal welfare weight than a wife with a low income husband. As a result, optimal tax schedules will not be separable because of the redistributive motives of the government.

We derive optimal tax rate formulas as a function of the labor supply elasticities, the strength of redistributive tastes of the government (measured by the curve social marginal welfare weights), and the distribution of earnings abilities in the population. We show how those optimal tax formulas can be obtained by considering small tax reforms around the optimum schedule, which allows us to understand the economic intuition behind each of the terms in the formulas and how the formulas relate to the classical individualistic optimal income tax theory. Our results show that the marginal tax rate faced by the primary earner at a given ability level and averaging over secondary earners is identical to the marginal tax rate obtained in the standard individualistic Mirrlees model. Thus, the presence of the secondary earner introduces heterogeneity in marginal tax rates for primary earners (depending on the earnings of the spouse) but without affecting the average.

We then analyze the asymptotics of optimal tax formulas when the earnings of the primary earner are large. Strikingly, for a wide class of social welfare objectives, we show that the tax distortion on the secondary earner vanishes asymptotically when the earnings of the primary earner become large. In other words, the earnings of spouses married to very high income husbands should be exempted from income taxation. The intuition for this surprising result can be seen by considering the following two possible tax reforms.

First, increasing the tax on couples with two earners and high income husbands requires to increase the marginal tax on primary earners (with working spouses) at some level. Therefore,
such a tax creates negative labor supply responses both for the primary and secondary earner. Second, increasing the tax on couples with high income primary earners and non-working spouses requires to increase the marginal tax of primary earners (with non working spouses) at some level. This creates a negative labor supply response for primary earners but a positive labor supply response for secondary earners. However, couples with very high primary earner’s income have the same marginal social welfare weight, irrespective of the wife earnings, as the contribution of the secondary earner to family income is small and negligible. Thus, the first reform clearly is less desirable than the second one, showing that reducing the tax on secondary earners is always desirable up to the point where the tax vanishes and the secondary earner labor supply response becomes irrelevant for government finances.

Finally, we develop numerical simulations (IN PROGRESS) of a simple stylized model where secondary earners face a binary labor supply choice (working or not) and where primary earners differ in their earnings abilities and supply labor along the intensive margin as in the Mirrlees (1971) model. We calibrate the preference parameters so that the magnitude of the labor supply responses of primary and secondary earners match those found in empirical studies. Our simulations show that the tax rate on secondary earners is positive and actually decreasing with the earnings of the primary earner (and vanishes to zero asymptotically as our theoretical result implied). Therefore, in contrast to the redistributive principle described above, wives with higher income husbands are taxed less than wives with lower income husbands. As a result, the marginal tax rate faced by the primary earner is actually lower when the secondary earner works. This striking result can be understood as follows.

Conditional of the earnings of the primary earner, the government would like to redistribute from two earner couples toward one earner couples. The value of such redistribution is larger for couples with low primary earnings because the contribution of the secondary earner to welfare is then more important. Therefore, the redistributive virtue of taxing secondary earnings is actually higher at the bottom on the primary earnings distribution and vanishes to zero at the top, which explains our counter-intuitive findings.

In the model, redistribution from couples with high primary earnings toward couples with low primary earnings takes place according to the logic of the standard Mirrlees (1971) model. As we discussed above, when the primary earnings are very high, the contribution of secondary earner becomes negligible and the value of such redistribution vanishes.
As mentioned above, the average marginal tax rate conditional on primary earnings is indeed identical to the one obtained in the Mirrlees model. Redistribution conditional on primary earnings takes place by transferring income from two earner couples toward one earner couples. Such a transfer creates a tax wedge on secondary earnings. This tax wedge is largest at the bottom because this is where redistribution from two earner couples toward one earner couples is the most valuable. The tax wedge goes to zero when primary earnings grow because redistribution from two earner couples toward one earner couples becomes valueless in the limit.

Thus, although our results seem surprising at first sight, they obey a simple redistributive logic. If the tax schedule for two earner couples is seen as the base schedule. The tax schedule for one earner couples is then obtained from that base schedule by giving a larger tax break to those couples with low primary earnings than for those with large primary earnings. In the limit where primary earnings go to infinity, the tax break is zero. This shrinking tax break generates of course an implicit tax on secondary earners which decreases with primary earnings.\footnote{This logic is actually similar to the redistributive logic of the two type optimal income tax model of Stiglitz (1982): redistribution from the high skilled toward the low skilled imply that difference in after-tax incomes will be smaller than the difference is before-tax incomes. As there is no distortion for the high skilled, such a redistribution can only be achieved by imposing a marginal tax on low skilled workers.}

The paper is organized as follows. Section 2 describes the general model. Section 3 presents the case where secondary earners respond only along the extensive margin. Section 4 shows how our results are modified when secondary earners respond along the intensive margin. Section 5 proposes numerical simulations of the theoretical models using empirical elasticities from the labor supply literature (IN PROGRESS). Finally, Section 6 offers a brief conclusion.

2 The General Model

2.1 Couples Labor Supply Modelling

We will consider a fully rational and efficient joint labor supply decision model, whereby each couple is characterized by a single utility function of the form \( u(c, z_p, z_s) = c - h(z_p, z_s) \) where \( z_p \) and \( z_s \) denote the earnings of primary and secondary earner and \( c \) is consumption (or disposable...
income) defined as earnings net of income taxes and transfers. We denote by \( T(z_p, z_s) \) the net income tax or transfer for a couple with earnings \((z_p, z_s)\) and hence \( c = z_p + z_s - T(z_p, z_s) \). The function \( h(z_p, z_s) \) represents the costs of working and earning \((z_p, z_s)\) for the couple and will be specified in detail later on. We always assume for tractability that the function \( h(z_p, z_s) \) is separable between \( z_p \) and \( z_s \) (so that increasing one’s labor supply does not affect the other’s cost of supplying labor).  

**Assumption 1** Couples utilities will take the following form:

\[
    u(c, z_p, z_s) = c - h_p(z_p) - h_s(z_s). 
\]

where \( h_p(z_p) \) and \( h_s(z_s) \) denote the disutility of earning \( z_p \) and \( z_p \) for the primary and secondary earner respectively.

This quasi-linear utility specification amounts to ruling out income effects in the labor supply decisions of both earners. We make this assumption for two main reasons. First, as is well known from the Industrial Organization literature on nonlinear pricing (see e.g., Wilson, 1993) and as shown more recently in Diamond (1998) in the context of the Mirrlees model of optimal income taxation, ruling out income effects simplifies substantially the theoretical analysis, and makes the problem tractable. Second, the large empirical literature on labor supply estimates in general small income effects (see e.g., Blundell and MaCurdy, 1999) and therefore considering the case with no income effects is perhaps a useful benchmark.

The couple chooses \((z_p, z_s)\) so as to maximize its utility subject to its budget constraint \( c = z_p + z_s - T(z_p, z_s) \). It is important to note that our model is equivalent to a single decision maker optimizing along two dimensions (the earnings of the primary earner and the earnings of the secondary earner) and thus there is no conflict within the couple about consumption or labor supply decisions. This is in contrast to the recent literature following Chiappori (1992) which models couples as two individual utility maximizers interacting with each other. We adopt the single decision maker hypothesis for simplicity\(^5\) and because it may provide a useful

\(^4\)In practice, if there are economies of scale in home production, such as child care, such a separability assumption would not be met. We consider the case of separable disutility of effort as a useful and simple benchmark and discuss in conclusion how relaxing that assumption would affect our results.

\(^5\)How joint taxes \( T(z_p, z_s) \) are “shared” among members of the couple is a critical and difficult issue to tackle in the Chiappori collective model.
benchmark. However, it would certainly be useful to know how our results would be affected if we were to use a more sophisticated and realistic model of joint decision making.

2.2 The Government Optimization Problem

As in standard optimal income tax models, the government maximizes a social welfare function defined as a (weighted) sum of the couples utility functions perhaps suitably transformed with a concave and increasing function. The weights and the concave transformation reflect the government’s tastes for redistribution. If we index by $m \in M$ (a possibly multi-dimensional set of measure 1), the couples utility functions $u^m(c, z_p, z_s)$ and denote by $d\nu(m)$ the density measure of couples, $\delta^m$ the weight on couple $m$, and $\Psi(.)$ the concave and increasing social welfare function, total welfare can be written as:

$$W = \int_M \delta^m \Psi(u^m(z_p + z_s - T(z_p, z_s), z_p, z_s))d\nu(m). \quad (2)$$

We choose to model social welfare using both weights $\delta^m$ and the concave transformation $\Psi(.)$ in order to encompass a wide range of possibilities which we will specify below. The government chooses the tax function $T(.)$ to maximize $W$ subject to two constraints. First, for a given tax function $T(.)$, each couple chooses $(z_p, z_s)$ to maximize its utility. Second, the government must satisfy a budget constraint of the form:

$$\int_M T(z_p, z_s)d\nu(m) \geq E, \quad (3)$$

where $E$ represents exogenous spending per family (such as public goods funding, the cost of administering the tax and transfer system and other government operations, etc.). It is important to keep in mind that $T(.)$ represents both taxes and transfers. From now on, we will denote by $\lambda$ the multiplier of the government budget constraint (3).

A tax system $T(z_p, z_s)$ is defined as separable if and only if it can be written as $T(z_p, z_s) = T_p(z_p) + T_s(z_s)$

Hence, a tax system is separable if and only if the cross partial derivative is zero, that is, one earner marginal tax rate is independent of his or her spouse’s earnings. A fully joint tax system as the federal individual income tax in the United States is characterized by $T(z_p, z_s) =$
$T(z_p + z_s)$. The main goal of the paper is to characterize the optimal function $T(\cdot)$ when no restrictions are imposed on the cross partial derivatives.

A useful concept representing the redistributive tastes of the government are the marginal social welfare weights which are defined as the social marginal value of an extra dollar of consumption given to a given couple $m$ and expressed in terms of the value of public funds. Hence, as utilities are quasi-linear in consumption (so that $u^m_c = 1$), the social marginal welfare weight for couple with characteristics $m$ is simply defined as $g^m = \delta^m \Psi(u^m)/\lambda$. With no income effects, a marginal dollar of public funds is valued as much as an additional dollar redistributed uniformly to all couples (a set of measure one) and therefore

$$\int_M g^m d\nu(m) = 1.$$ \hspace{1cm} (4)

Equation (4) shows that social marginal welfare weights average to one in the population. They are higher than one for those relatively disadvantaged (toward whom the government would like to redistribute) and below one for those relatively advantaged (whom the government would like to tax).

The first question we want to investigate is under which conditions a separable tax system of the form $T(z_p, z_s) = T_p(z_p) + T_s(z_s)$ is optimal. We can obtain the following result.

**Theorem 1** The couples utility functions are separable of the form $u^m(c, z_p, z_s) = c - h^{m_p}_{z_p}(z_p) - h^{m_s}_{z_s}(z_s)$, where index $m = (m_p, m_s)$ characterizes each couple and $m_p$ and $m_s$ are distributed independently.

If the welfare weights $\delta^m$ are separable such that $\delta^m = \delta^{m_p}_{z_p} + \delta^{m_s}_{z_s}$ and $\Psi(u) = u$ (so that the social marginal welfare weights are also separable $g^m = g^{m_p}_{z_p} + g^{m_s}_{z_s}$), then the optimal tax schedule is separable and each component $T_p$ and $T_s$ is obtained by maximizing an individualistic optimal income tax problem.

**Proof:** See appendix.

This theorem shows that when three independence or separability assumptions are met, then the optimal tax schedule is also separable. First, the disutility of work has to be separable across the two members of each couple. This assumption means that when one member works more, this does not affect the cost of working for the other member. Such an assumption would
be violated if, for example, spouses like to share leisure time together (if one works more, then leisure is less valuable for the other spouse).

Second, the characteristics $m_p$ and $m_s$ of each member of the couple have to be distributed independently. This assumption is of course violated if high skilled individuals tend to marry high skilled individuals. In that case, high spousal earnings might indicate higher skills and hence that information should be used in the optimal tax system producing a non-separable optimal tax schedule.

Last, the social welfare weights need to be separable between the two members of the couple. To understand better the meaning of this assumption, let us assume that $m_p$ and $m_s$ are scalar and represent ability of each member of the couple. By definition, the separability assumption means that giving a dollar to a couple produces a marginal welfare gain equal to $\delta_p^{m_p} + \delta_s^{m_s}$. It is useful to think that $\delta_p^{m_p}$ is the marginal welfare value for the primary earner and $\delta_s^{m_s}$ is the marginal welfare value for the secondary earner (of getting that single extra dollar for the couple). The separability assumption then means that the welfare value for a given individual is independent of the ability of his spouse. As a result, a low ability individual will receive the same welfare weight irrespective of his spouse ability.

If the government uses a standard welfare function with uniform weights $\delta^m$ and a concave transformation $\Psi(u^m)$ of utility, the marginal welfare weights $g^m = \Psi'(u^m)/\lambda$ will not be separable as giving to a low ability individual is less valuable when his spouse has high ability. Therefore, Theorem 1 shows that we should not expect separability of the optimal tax system in natural situations. This is precisely the issue we want to investigate from now on: what are the implications of imposing a standard redistributive social welfare criterion on the form of the optimal tax. In the rest of the paper, we impose more structure on the labor supply responses in order to derive optimal tax formulas and obtain stronger results on the shape of optimal tax schedules.

3 Extensive Response for Secondary Earners

3.1 Deriving Optimal Income Tax Rates

In this model, the primary earner is characterized by a scalar ability parameter $n$. The cost of earning $z_p$ for a primary earner with ability $n$ is $nh_p(z_p/n)$ (with a slight abuse of
notation relative to our previous notation \( h_p(z_p) \) where \( h_p \) is an increasing and convex function normalized such that \( h_p(0) = 0 \) and \( h'_p(1) = 1 \). The primary earner chooses \( z_p \) to maximize utility taking \( z_s \) as given. Thus \( z_p \) is chosen to maximize \( z_p - T(z_p, z_s) - nh_p(z_p/n) \) which leads to the primary earner’s first order condition:

\[
h'_p(z_p/n) = 1 - T'_p,
\]

where \( T'_p \), the partial derivative of \( T(.) \) with respect to \( z_p \), is the marginal tax rate of the primary earner. Note that in the case of no tax distortion, \( T'_p = 0 \), our normalization assumption \( h'_p(1) = 1 \) implies that \( z_p = n \): primary earnings would be identical to ability \( n \). Hence, it is useful to interpret \( n \) as potential earnings. Positive marginal tax rates depress actual earnings \( z_p \) below potential earnings \( n \). If the cross partial derivative of the tax function is non zero, then \( T'_p \) will depend on the secondary earner’s labor supply decision. In this case, there will be an interdependence between the two spouses labor supply decision.

We classically define the elasticity of earnings with respect to the net-of-tax rate (one minus the marginal tax rate) as:

\[
\varepsilon = \frac{1 - T'_p}{z_p} \cdot \frac{\partial z_p}{\partial(1 - T'_p)} = \frac{nh'_p(z_p/n)}{z_p h''_p(z_p/n)}.
\]

Because we assume away income effects, the compensated and uncompensated elasticity of labor supply are of course identical.

In the simplest version of the model we first consider, the secondary earner is characterized by a fixed cost of work parameter \( q \). We denote by \( l = 0,1 \), the binary labor supply decision of the secondary earner. If the secondary earner works, she incurs a cost \( q \) and earns a fixed amount \( w \) that is uniform across couples. Hence the earnings of the secondary earner can be written as \( z_s = wl \). The utility function of a couple with parameters \((n,q)\) is:

\[
u = z_p + wl - T(z_p, wl) - nh_p(z_p/n) - q \cdot l,
\]

We assume that the couple characteristics \((n,q)\) are distributed according to a density distribution defined over \([\underline{n}, \bar{n}] \times [0, \infty)\). We denote by \( P(q|n) \) the cumulated distribution function of \( q \) conditional on \( n \), \( p(q|n) \) the density of \( q \) conditional on \( n \), and \( f(n) \) the unconditional density of \( n \), so that the density of the joint distribution of \((n,q)\) is given by \( p(q|n) \cdot f(n) \).
Because the secondary earner’s labor supply decision is binary and there are only two earnings outcomes possible (0 or w), the tax schedule $T(.)$ is simply characterized by two one-dimensional tax schedules faced by the primary earner: $T(.,0)$ when his spouse does not work and $T(.,w)$ when his spouse works which we will denote by $T(.,0)$ and $T(.,1)$ respectively to economize on notation.

Let us denote by $z_p(n,1)$ and $z_p(n,0)$, the optimal earnings of a primary earner with productivity $n$ conditional on spouse participation and non-participation, respectively. Those earnings supply functions depend of course on the tax schedule.

For the secondary earner to enter the labor market and work, the utility from participation must be greater than or equal to the utility from non-participation. Let us denote by $V(n,0) = z_p - T(z_p,0) - nh_p(z_p/n)$, the indirect utility of the couple when the spouse is not working and by $V(n,1) = z_p + w - T(z_p,1) - nh_p(z_p/n)$ the indirect utility (excluding the fixed costs of work) when the spouse is working. The envelope theorem implies that:

$$V_0'(0) = -h_p(z_p(0)/n) + (z_p(0)/n)h_p'(z_p(0)/n)$$
$$V_0'(1) = -h_p(z_p(1)/n) + (z_p(1)/n)h_p'(z_p(1)/n)$$

where we have dropped the index $n$ whenever there is no ambiguity. The participation constraint may then be written as

$$q \leq V(n,1) - V(n,0). \quad (8)$$

For families with a fixed cost below the threshold-value $\bar{q} = V(n,1) - V(n,0)$, the secondary earner works. For families with a fixed cost above the threshold the secondary earner stays out of the labor force. If the tax function is not separable, the value of $\bar{q}$ and hence the participation decision of the secondary earner will depend on the labor supply decision of the primary earner. The probability of labor force participation of the secondary earner for a given ability level $n$ of the primary earner is given by $P(\bar{q}|n)$.

We state and analyze formally the optimization program of the government in appendix and obtain the following optimal tax formulas:

**Proposition 1** The first order conditions for the optimal marginal tax rates $T_p'(1)$ and $T_p'(0)$ at ability level $n$ can be written as follows,
In equations (9) and (10), all the terms outside the integral are evaluated at ability level \( n \) and all the terms inside the integral are evaluated at \( n' \). \( g(n',0) \) and \( g(n',1) \) are the average marginal welfare weights for couples with primary earners’ ability \( n' \) and secondary earners not working and working respectively.

**Heuristic Proof of Proposition 1**

In order to understand the economic intuitions behind the formulas in Proposition ??, it is useful to provide a heuristic derivation of those formulas based on the analysis of a small tax reform around the optimum schedule.

It is a useful first step to recall briefly the derivation of the optimal tax rate formula in the standard individualistic case (with no secondary earner). In that case, the model is a classic optimal income tax model à la Mirrlees (1971) with no income effects as in Diamond (1998). The heuristic derivation of optimal income tax rates has been developed by Piketty (1997) and Saez (2001).

Suppose, as illustrated on Figure 1, that we increase the income tax by \( dT \) for individuals with ability above \( n \). This increase in taxes is obtained through a small increase \( d\tau \) in the marginal tax rate in the small band of ability levels \([n, n + dn]\). This tax reform raises more tax revenue from all taxpayers above the small band but decreases their utility. The gain for the government net of the welfare cost is

\[
dG = dT \cdot \int^n \left[ 1 - g(n') \right] f(n')dn',
\]

where \( g(n') \) is the marginal social welfare weight for individuals with ability \( n' \), and \( f(n') \) is the density distribution of ability.
In the small band \([n, n + dn]\), there is a reduction in earnings due to the higher marginal tax rate. This decreases tax revenue collected from those taxpayers. The total tax revenue lost due to this behavioral response is proportional to the density of taxpayers \(f(n)\), the size of the labor supply elasticity \(\varepsilon\), and the marginal tax rate \(T'\) in the small band. More precisely, we can show the loss in tax revenue is

\[
dL = -dT \cdot nf(n) \cdot \varepsilon \cdot \frac{T'}{1 - T'}.
\]

At the optimum, such a reform cannot increase welfare and hence the sum of the loss in tax revenue \(dL\) and the net gain \(dG\) must be zero implying the optimal income tax rate formula:

\[
\frac{T'}{1 - T'} = \frac{1}{\varepsilon} \cdot \frac{1}{nf(n)} \cdot \int_n^n [1 - g(n')]f(n')dn'. \tag{11}
\]

Let us now examine how the introduction of the secondary earner modifies equation (11). In the case of the secondary earner, the tax system can be depicted as on Figure 2 as a pair of tax schedules, one for couples with working spouses and one for couples with non-working spouses. Note that the vertical difference between the two schedules, \(T(1) - T(0)\), is the extra tax paid by the couple when the secondary earner enters the labor force.

Let us consider, as displayed in dashed line on Figure 2, the same reform as before but only for couples with working spouses. More precisely, all couples with ability above \(n\) and with a working spouse face a small tax increase \(dT\) which is created by increasing the marginal tax rate in the small band \([n, n + dn]\). As above, this tax reform raises more tax revenue from all two earner couples above the small band but decreases their utility. The gain for the government net of the welfare cost is therefore

\[
dG = dT \cdot \int_n^n [1 - g(n', 1)]f(n')P(\bar{q}|n')dn',
\]

where \(g(n', 1)\) is the average marginal social welfare weight for couples with ability \(n'\) and a working spouse and \(P(\bar{q}|n)\) is the fraction of couples with ability \(n'\) for which the secondary earner works (those with fixed cost of work below the cut-off level \(\bar{q} = V(n', 1) - V(n', 0)\) as we explained above).

As above, the increase in marginal tax rate in the small band creates a negative labor supply response for the primary earner which reduces tax collected by

\[
dL = -dT \cdot P(\bar{q}|n) \cdot nf(n) \cdot \varepsilon(1) \cdot \frac{T_p'(1)}{1 - T_p'(1)},
\]
where $P(\bar{q}|n)$ is the fraction of couples with ability $n$ for which the secondary earner works and $\varepsilon(1)$ is the intensive elasticity of earnings with respect to the net-of-tax rate at ability level $n$ for primary earners with ability $n$ in two earner couples.

However, in contrast to the previous case, there is now an additional effect as the tax reform will induce some working spouses (with primary earner above $n$) to drop out of the labor force and fall back on the non-working spouse schedule (as illustrated on Figure 2). At ability level $n'$, those couples with fixed costs of working between $\bar{q}$ and $\bar{q} - dT$ (there are $p(\bar{q}|n') \cdot f(n') \cdot dT$ of those couples) will move to the non-working spouse schedule, creating a tax loss for the government of $-[T(1) - T(0)] \cdot p(\bar{q}|n') \cdot f(n')dT$. Hence the total tax loss due to this participation behavioral response is

$$dP = -dT \cdot \int_{n}^{\bar{n}} \left[ T(1) - T(0) \right] \cdot p(\bar{q}|n') \cdot f(n')dn'.$$

At the optimum, the sum of the three effects $dG$, $dB$, and $dP$ should be zero which leads immediately to equation (10) in the Proposition.

Equation (9) can be obtained in the same way by considering an increase in the tax for the couples above $n$ with non-working spouses. In that case, the participation effect goes in the opposite direction: some non-working spouses are induced to start working, increasing government tax revenue. As a result, the participation term in equation (9) appears with a positive sign.

### 3.2 Properties Optimal Income Tax Rates

- **Comparing $T_p'(1)$ and $T_p'(0)$**

  It is useful to note first that the average marginal tax rate over one and two earner couples is exactly the same of the marginal tax rate of the individualistic standard theory reported in (11). By taking the weighted sum of (9) and (10), we obtain,

  $$\varepsilon(0)(1 - P(\bar{q}|n)) \cdot \frac{T_p'(0)}{1 - T_p'(0)} + \varepsilon(1)P(\bar{q}|n) \cdot \frac{T_p'(1)}{1 - T_p'(1)} = \frac{1}{nf(n)} \cdot \int_{n}^{\bar{n}} [1 - \bar{g}(n')]f(n')dn',$$

  where $\bar{g}(n')$ is the average social marginal welfare weights for couples with ability $n'$. This result can be obtained heuristically by increasing slightly the tax for all couples with ability above $n$. In that case, there is no change in the participation decision of secondary earners.
and therefore the only behavioral response is a substitution effect for primary earners around $n$. Thus, this result shows that redistribution from high to low primary earners’ ability follows exactly the same logic as in the standard Mirrlees (1971) optimal income tax model. The introduction of the secondary earner does not change the average marginal tax rate faced by primary earners but introduces a difference in the marginal tax rate faced with one versus two earner couples, which we now examine in detail.

There are two main differences in the expressions for $T_p(1)$ and $T_p(0)$ obtained in Proposition 1.

First, for a given ability level $n'$, couples with a working spouse will have a higher level of consumption and utility than couples with a non-working spouse. Therefore, if the government values redistribution, the corresponding social marginal weights should be lower for two earners couples than for one earner couples: $g(n', 1) < g(n', 0)$. Proposition 1 then shows that this should tend to make the marginal tax rate in two earners couples $T_p(1)$ higher than the marginal tax rate in one earner couples $T_p(0)$. This corresponds to the standard intuition that two earner couples are better off and hence should face higher tax rates for equity reasons.

Second, in the normal case where the secondary earner faces a tax (i.e., $T(1) - T(0) > 0$, a point we examine in detail below), the participation effect decreases $T_p(1)$ and symmetrically increases $T_p(0)$. This is because reducing the tax on two earner couples or increasing the tax on one earner couples attracts secondary earners to the labor force and hence creates a positive tax revenue externality.

Therefore, the secondary earner generates an efficiency-equity tradeoff: for equity reasons, the tax on two earner couples should be higher than on one earner couples but reducing the tax on two earner couples relative to one earner couples increases labor force participation and hence the efficiency of the tax system.

- **Classical zero top and bottom results**

Sadka (1976) and Seade (1977) demonstrated one of the most striking property of the Mirrlees (1971) model, namely that the marginal tax rate should be zero at the top and at the bottom (provided the bottom skill is positive and everybody works). The same property holds in the two earner model we are considering.
Proposition 2 If the distribution of abilities $n$ is bounded, then $T_p(0) = T_p(1) = 0$ at the top $\bar{n}$.

If the bottom skill is positive then $T_p(0) = T_p(1) = 0$ at the bottom $\bar{n}$.

This proposition is a direct consequence of the transversality conditions (see Appendix). It is easy to see why they hold using the heuristic variational method described above. Let us come back to Figure 2 and assume that the increase in marginal tax rate took place at the very top in the small band $[\bar{n} - dn, \bar{n}]$. In that case, the mechanical effect (net of the welfare cost) is negligible relative to the primary earner labor supply effect because there is nobody above $\bar{n}$ to collect the extra taxes $dT$ from. Similarly the participation effect is negligible relative to the primary earner intensive labor supply effect. Thus, the first order condition holds only if $T_p(1) = 0$. The same proof can be adapted to the bottom case as well.

Numerical simulations in the context of the Mirrlees model (see e.g., Tuomala, 1990) have shown that the top result is not of much use in practice because it is true only at the very top and hence applies only to the top earner. Top tails on the earnings distributed are very well approximated by Pareto distributions and thus it is much more fruitful to consider infinite tails to obtain useful high income optimal income tax results (see for example Saez, 2001). We therefore now turn to this case.

• Asymptotic $T(1) - T(0)$

Suppose that $\bar{n} = \infty$ so that the ability distribution of primary earners has an infinite tail. In that case, we expect that, for all reasonable welfare functions, $g(n, 0)$ and $g(n, 1)$ will converge to the same value $g^\infty$, because the extra income generated by the secondary earner becomes negligible relative to the primary earner income. In the case where $g^\infty = 0$, the optimal tax system extracts the most tax revenue from the very rich (soak the rich). We also assume that the primary earner elasticity $\varepsilon$ converges to an asymptotic value $\varepsilon^\infty$ when $n$ goes to infinity.

Top tails of income distributions are very well approximated by Pareto distributions (see Saez (2001) for a discussion of this point and its consequences for optimal income taxation). Therefore, let us assume further that in the top tail, the distribution of abilities $n$ is Pareto distributed with Pareto parameter $a$ and that fixed costs of work $q$ are distributed indepen-
ently of \( n \) at the top with distribution \( P(q) \) (using the same notation as above). Under those assumptions, we can prove the following result.

**Proposition 3** \( T(1) - T(0) \) converges to zero as \( n \) goes to infinity, so that the tax on secondary earners goes to zero as the earnings of the primary earner increase to infinity. As in the Mirrlees model, the optimal marginal tax rates for primary earners \( T_p'(0) \) and \( T_p'(1) \) both converge to \( (1 - g^\infty)/(1 - g^\infty + a \cdot \varepsilon^\infty) \) as earnings go to infinity.

The formal proof of this proposition is easy to obtain by looking at the asymptotics of equations (9) and (10) and in presented in appendix. The result in Proposition 3 is rather striking, the earnings of spouses of high income earners should be exempted from taxation, even in the case where the government tries to extract as much tax revenue from high income couples as possible (case \( g^\infty = 0 \)).

Let us try to understand the economic intuition behind this surprising result. Let us assume that \( T(1) - T(0) \) converges to some limit \( \Delta T > 0 \) as depicted on Figure 3. For large \( n \), a constant fraction \( P(\bar{q}) \) of couples have two earners. Consider then increasing the tax on one earner couples and decreasing the tax on two earner couples above some high ability level \( n \) as shown on Figure 3. The increase in tax for two earner couples is \( dT/P(\bar{q}) \) and the decrease in tax for one earner couples is \( -dT/(1 - P(\bar{q})) \) so that the net effect on taxes collected (absent any behavioral response) is zero. Importantly, the direct welfare effect is also zero because the reduced welfare of one earner couples is exactly compensated by the increased welfare of two earner couples as the social marginal welfare weights are identical (and equal to \( g^\infty \)) for both groups.

These tax changes are obtained by raising (decreasing) the marginal tax rate for one earner (two earner) couples around \( n \). Those marginal tax rate changes produce behavioral responses going in opposite direction for one earner and two earner couples and these two effects offset each other exactly. As a result, the net fiscal effect of those behavioral responses is also zero.

Finally, the tax change induces a number of non-working spouses above \( n \) to join the labor force. Each of these movers would pay \( \Delta T \) extra-taxes and hence produce a positive fiscal

\[^6\]This result is reminiscent of the classical zero top result of Sadka and Seade described above but the logic is quite different. In the present case, the tax at the top is zero for the secondary earner while the tax is actually positive for the primary earner.
effect. This positive effect is the net total effect of the reform as all the previous effects cancel out. If $\Delta T > 0$, this reform is desirable for the government showing that this cannot be an optimum. If $\Delta T < 0$, the opposite tax reform would increase welfare. Therefore we must have $\Delta T = 0$ asymptotically as stated in Proposition 2.

In summary, imposing a tax on secondary earners with high income spouses (above $n$) requires to increase the marginal tax on the primary earners at level $n$, creating an additional negative labor supply response. This negative labor supply response can be compensated by a reduction in the marginal tax rate on single earner couples with ability $n$, creating a subsidy for one earner couples (relative to two earner couples) above $n$. As shown on Figure 3, this amounts to redistributing from two-earner couples to one-earner couples. For couples with large primary earner incomes, there is no value in such a redistribution as marginal welfare weights for one and two earner couples are about the same. As a result, there is no point in imposing a tax on the secondary earner.

- **Conjectures on the General Case**

Simple results:

**Lemma 1** The transversality condition implies

\[
\int \bar{n} \left[ T(1) - T(0) \right] p(\bar{q}|n') f(n') dn' = \int \bar{n} \left[ g(0) - 1 \right] \cdot [1 - P(\bar{q}|n')] \bar{f}(n') dn' = \int \bar{n} \left[ 1 - g(1) \right] \cdot P(\bar{q}|n') f(n') dn'.
\]

Therefore, the tax on secondary earners $T(0) - T(1)$ is positive on average if two earner couples have on average marginal welfare weights lower than one or equivalently (as the average marginal welfare weight for the full population is one) if one earner couples have on average marginal welfare weights higher than one.

For a given ability level, two earner couples are better off than one earner couples (as their fixed cost of working is smaller, two earner couples always have the option to become one earner couples and get the utility of one earner couples). As a result, for any redistributive social welfare function $g(0, n) > g(1, n)$ for all $n$.

However, this does not necessarily imply that the average marginal welfare weight of one earner couples is larger than one because of composition effects. For example, if high ability
couples had very large fixed costs of work and low ability couples had small fixed costs of work, low ability couples could be better off than high ability couples if the secondary earnings \( w \) are large relative to the primary earnings ability.

We should be able to prove something of the form:

**Conjecture 1** Assume that \( q \) and \( n \) are distributed independently and that the earnings elasticity, \( \varepsilon \), is constant. Then:

\[ T(1) - T(0) \text{ is positive for all ability levels } n \text{ and is decreasing in } n. \]

For a reasonable welfare function, we would expect \( g(n,0) - g(n,1) \) to be positive, and decreasing in \( n \), and eventually converging to zero when \( n \) grows (as spouses earnings become negligible). In that case, we would expect \( T(1) - T(0) \) to be positive and decreasing to zero as \( n \) increases and becomes large. This suggests that redistribution toward couples with only one earner takes the reform of a non-working spouse allowance which decreases with the primary earner’s earnings, and is completely phased-out asymptotically. This allowance creates a tax on spouses \( T(1) - T(0) > 0 \). Interestingly, the spouses of the low skilled are implicitly taxed because this is the only (costly) way to redistribute toward low skilled couples with only one earner.

The fact that \( T(1) - T(0) \) is decreasing in \( n \) should imply that \( T'_p(1) > T'_p(0) \).\(^7\) Thus, this spousal taxation also generates higher marginal tax rates on primary earners with non working spouses because the allowance is phased out as earnings increase.

We can prove the following simpler result on necessary and sufficient conditions to get an optimum tax that is separable in the earnings of the each member of the couple. This result can be seen as a Corollary to the much more general Theorem 1. In the present context, separability is equivalent to having \( T(1) - T(0) \) independent of \( n \).

**Lemma 2** Let us assume that \( q \) is distributed independently of \( n \) and with distribution \( P(q) \) and density \( p(q) \).

If \( g(0) - g(1) \) is constant over \( n \), then

\(^7\)This implication is not trivial because the derivatives of \( z_p(1) \) and \( z_p(0) \) with respect to \( n \) are not equal.
\[ T(1) - T(0) = (g(0) - g(1)) \cdot \frac{P(\bar{q})(1 - P(\bar{q}))}{p(\bar{q})}. \]

Furthermore both \( T(1) - T(0) \) and \( \bar{q} \) are independent of \( n \) and hence \( T_p'(0) \equiv T_p'(1) \).

Conversely, if the optimum is such that \( T(1) - T(0) \) is independent of \( n \), then it must be the case that \( g(0) - g(1) \) is constant over \( n \).

Proof: For the first part, if we assume that \( g(0) - g(1) \) is constant then it is easy to check that \( T(1) - T(0) \) constant (and as in the Proposition) satisfies the first order conditions of Proposition 1. Note that \( T(1) - T(0) \) constant implies that \( z_p(1) = z_p(0) \) (as the labor supply of the primary earner is independent of the secondary earner participation decision), and hence \( \bar{q} = w - T(1) + T(0) \) is also independent of \( n \).

For the second part, if \( T(1) - T(0) \) is constant, then \( T_p'(0) = T_p'(1) \) and \( z_p(1) = z_p(0) \) for all \( n \). Thus, \( \bar{q} = w - T(1) + T(0) \) is also independent of \( n \), and therefore, Proposition 1 implies that, for every \( n \),

\[
\int_{n} \frac{[1 - g(0)] + [T(1) - T(0)]}{1 - P(\bar{q})} \frac{p(\bar{q})}{P(\bar{q})} \cdot f(n')dn' = \int_{n} \frac{[1 - g(1)] - [T(1) - T(0)]}{P(\bar{q})} \frac{p(\bar{q})}{P(\bar{q})} \cdot f(n')dn'.
\]

Taking the derivative of this expression with respect to \( n \) then implies that \( g(1) - g(0) \) is independent of \( n \).

This result shows that, in order to generate a constant tax wedge for the secondary earner, a constant difference in the social marginal welfare weight between one and two earner couples conditional on ability is required. Of course, such a constant difference cannot be obtained with standard social welfare functions with uniform weights \( \delta^m = 1 \) and a concave transformation \( \Psi(u) \).

This result, nevertheless, offers a useful benchmark. In the normal case, as we discussed above, we expect \( g(0) - g(1) \) to be positive and decreasing to zero as \( n \) increases. The bench-

\[ ^{8} \text{Note that the elasticity } \varepsilon \text{ will also be the same for one and two earner couples conditional on the ability level } n. \]

\[ ^{9} \text{With such a social welfare functions, } g(0) - g(1) \text{ constant is approximately true if } w \text{ is very large relative to } n. \]

Interestingly, in that case, we could obtain the converse result of Proposition 2, namely that \( T_p'(0) = T_p'(1) \simeq 0 \). When one earner dominates so that the other earner does not affect the marginal social weight, there should be no distortion in the other earner labor supply decision. More on this below.
mark result then suggests that $T(1) - T(0)$ should also be decreasing (we know that it converges to zero from the asymptotic analysis).

We are in the process of doing numerical simulations of this model and see whether the results $T(1) - T(0) > 0$ and $T(1) - T(0)$ decreasing in $n$ are pervasive or hold only in restricted situations.

In order to make progress in our understanding of the shape of $T(0) - T(1)$, it useful to differentiate equations (9) and (10) with respect to $n$ and take their difference in order to obtain:

$$
[T(1) - T(0)] \cdot \frac{p(\bar{q})}{P(\bar{q})(1 - P(\bar{q}))} = g(0) - g(1) +
$$

$$
\left[ \varepsilon(1) \cdot \frac{T'_{p}(1)}{1 - T'_{p}(1)} \cdot (1 - P(\bar{q})) + \varepsilon(0) \cdot \frac{T'_{p}(0)}{1 - T'_{p}(0)} \cdot P(\bar{q}) \right] \cdot n \cdot p(\bar{q}) \cdot \frac{\partial \bar{q}}{\partial n} \cdot P(\bar{q})(1 - P(\bar{q})) +
$$

$$
\frac{1}{f(n) \cdot \partial n} \left[ \left( \varepsilon(1) \cdot \frac{T'_{p}(1)}{1 - T'_{p}(1)} - \varepsilon(0) \cdot \frac{T'_{p}(0)}{1 - T'_{p}(0)} \right) \cdot n f(n) \right],
$$

This rather complicated equation can be seen as a second order differential equation in $T(1) - T(0)$. The constant term $g(0) - g(1)$ is the key element creating a non-zero difference between $T(1)$ and $T(0)$. If $g(0) \equiv g(1)$, then it is clear that $T(0) \equiv T(1)$ will solve this difference equation (in that case $\bar{q}$ is independent of $n$ and hence $\partial \bar{q}/\partial n = 0$ and the optimum schedule $T(0)$ will be exactly as in Diamond (1998).

In the general case $g(0) - g(1)$ is decreasing in $n$. The first term in equation (13) suggests that this creates a tax wedge for the secondary earner which should be decreasing. The remaining terms in equation (13) are due to the complex effects that such a wedge introduces on the marginal tax rate and labor supply responses of the primary earner. Still, we would intuitively expect that those effects would at best attenuate but no reverse the direct effect due to $g(0) - g(1)$. That is why, we expect $T(1) - T(0)$ to be decreasing for a wide range of situations.

In sum, redistribution from couples with high primary earnings toward couples with low primary earnings takes place according to the logic of the standard Mirrlees (1971) model. As mentioned above, the average marginal tax rate conditional on primary earnings is indeed identical to the one obtained in the Mirrlees model. Redistribution conditional on primary earnings takes place by transferring income from two earner couples toward one earner couples.
Such a transfer creates a tax wedge on secondary earnings. This tax wedge is largest at the bottom because this is where redistribution from two earner couples toward one earner couples is the most valuable \((g(0) - g(1))\) is highest). The tax wedge goes to zero when primary earnings grow because redistribution from two earner couples toward one earner couples becomes valueless in the limit \((g(0) - g(1))\) is zero in the limit \(n = \infty\).

Thus, although our results seem surprising at first sight, they obey a simple redistributive logic. If the tax schedule for two earner couples is seen as the base schedule. The tax schedule for one earner couples is then obtained from that base schedule by giving a larger tax break to those couples with low primary earnings than for those with large primary earnings. In the limit where primary earnings go to infinity, the tax break is zero. This shrinking tax break generates of course an implicit tax on secondary earners which decreases with primary earnings.

4 General Labor Supply Response for Secondary Earners

In the previous model, we considered a very simple extensive labor supply model for the primary earner. We demonstrated that the tax on secondary earners goes to zero when the earnings of the primary earner become large and that the tax system is separable if and only if the marginal welfare weights are also separable. Those two results are robust to the introduction of more complex labor supply responses for the secondary earner.

4.1 Discrete Model

It is possible to generalize the simple binary labor supply model for secondary earners that we presented above and consider that secondary earners have the option to choose among \(I + 1\) occupations denoted by \(i = 0, 1, ..., I\) and paying wages \(w_0 < w_1 < ... < w_I\). We assume further that occupation 0 is being out of the labor force and hence pays no wage \((w_0 = 0)\). Each secondary earner is characterized by a \(I + 1\) vector \(q = (q_0, ..., q_I)\) representing the costs associated with each occupation. We also adopt the normalization \(q_0 = 0\) (it is costless to be out of the labor force). For example, skilled individuals will have relatively low cost of working in the high paying occupations while low skilled individuals might have very large costs (perhaps infinite) of working in those occupations. Such a discrete occupational choice
model was used in Saez (2001) and is general enough to encompass most standard models of labor supply (intensive or extensive margins, etc.) by imposing restrictions of the distribution of $q$ in the population.

The labor supply of the primary earner is modelled exactly as in Section 3. Thus, the tax schedule is now represented by $I + 1$ simple tax schedules $T(z_p, i)$ for $i = 0, ..., I$ depending on which occupation the secondary earner has chosen. For a given $i$, the primary earner with ability $n$ chooses $z_p$ to maximize $z_p - T(z_p, i) - nh_p(z_p/n)$ so that:

$$h'_p(z_p(i)/n) = 1 - T'_p(i)$$

A above, we denote by $V(n, i) = z_p(i) + w_i - T(z_p(i), i) - nh_p(z_p(i)/n)$ the indirect utility of the couple (excluding the secondary earner cost of work $q_i$).

The secondary earner chooses the occupation $i = 0, ..., I$ which maximizes $V(n, i) - q_i$. Each couple in the population is now characterized by the parameters $(n, q)$. We denote again by $P(q|n)$ the distribution of $q$ in the population conditional on $n$ and by $F(n)$ the unconditional distribution of primary earners’ abilities (as above, we denote the density of distribution of $n$ by $f(n)$). For each $n$, let us denote by $Q_i$ the set of vectors $q$ for which occupation $i$ is optimal. Formally, $Q_i = \{q \text{ st } V(i) - q_i \geq V(j) - q_j \text{ for all } j\}$. so that it is a function of all the $V(j)$, $j = 0, ..., I$ and hence implicitly of the tax schedules. The measure of $Q_i$ with respect to the distribution $P(q|n)$ is denoted by $P_i(n)$ and is the fraction of secondary earners who choose occupation $i$ (conditional on ability $n$). Obviously, $P_i(n)$ depends on the tax schedule through the indirect utility levels $V(j)$ for $j = 0, ..., I$. We assume that the distribution $P(q|n)$ is regular enough so that the $P_i(n)$ are differentiable with respect to each $V(j)$.

**Lemma 3**

$$\sum_{j=0}^{I} P_j(n) = 1.$$ 

For all $i = 0, ..., I$, we have:

$$\sum_{j=0}^{I} \frac{\partial P_j(n)}{\partial V(i)} = 0,$$ (14)

$$\sum_{j=0}^{I} \frac{\partial P_i(n)}{\partial V(j)} = 0,$$ (15)
The first equation is obvious. The second follows from differentiating the first with respect to \( V(i) \). The third follows from the no income effects property: if all the \( V(j) \) are increased by the same amount, then the occupation choices stay identical for all couples, and as a result \( Q_i \) and \( P_i(n) \) do not change.

It is straightforward to generalize Proposition 1 as follows:

**Proposition 4** The first order conditions for the optimal marginal tax rate \( T'_p(i) \) at ability level \( n \) can be written as follows,

\[
\frac{T'_p(i)}{1 - T'_p(i)} = \frac{1}{\varepsilon(i)} \cdot \frac{1}{n f(n) P_i(n)} \int_n^{\bar{n}} \left\{ [1 - g(n', i)] P_i(n') + \sum_{j=1}^{I} (T(j) - T(0)) \frac{\partial P_j(n')}{\partial V(i)} \right\} f(n') dn',
\]

(16)

In equation (16), all the terms outside the integral are evaluated at ability level \( n \) and all the terms inside the integral are evaluated at \( n' \).

\( g(n', i) \) is the average marginal welfare weights for couples with primary earners’ ability \( n' \) and secondary earners choosing occupation \( i \).

The intuition for the result can be seen as follows. If the tax for occupation \( i \) is increased by \( dT \) above \( n \), then we have the welfare effect and the primary earner labor supply response (as above). This change decreases \( V(i) \) above \( n \) by \( dT \) (while all the other \( V(j) \) are kept constant). As a result, some secondary earners will switch to a different occupation. The net number of secondary earners at ability level \( n' \) leaving occupation \( j \) is \(-dT \cdot \partial P_j(n') \) partial \( V(i) \). Thus the net fiscal effect of all those occupational changes is:

\[
\sum_{j=0}^{I} T(j) \frac{\partial P_j(n')}{\partial V(i)} = \sum_{j=1}^{I} (T(j) - T(0)) \frac{\partial P_j(n')}{\partial V(i)} ,
\]

where the last equality is obtained using (14) from Lemma 3. This is exactly the expression inside the integral in equation (16).

As above, we obtain the classical transversality conditions at the bottom \( n_0 \) and the top \( \bar{n} \).

As above, we can analyze the asymptotics of equation (16) when \( n \) goes to infinity, and prove an analogous result as in Proposition ???. Namely, when \( n \) grows to infinity, for all \( j \),
$T(j) - T(0)$ converges to zero so that there is no tax distortion in the labor supply incentives of secondary earners asymptotically.

This result can be obtained as follows. We assume that marginal welfare weights $g(n, i)$ all converge to $g^\infty$ when $n$ goes to infinity. If we assume that all marginal tax rates $T^\mu_k(i)$ converge, they must converge to the same value $\tau^\infty$ (otherwise the tax on secondary earners would become infinitely large in absolute value, which cannot be optimal). Let us assume that $\Delta T(j) \equiv T(j) - T(0)$ converges to $\Delta T^\infty_j$. The $P_i(n)$ will also converge to $P^\infty_i$. If we assume that the top tail of $F(n)$ is Pareto with parameter $a$, then (16) can be rewritten asymptotically as:

$$
\frac{P^\infty_i}{1 - \tau^\infty} = \frac{1}{\varepsilon^\infty} \cdot \frac{1}{a} \left\{ P^\infty_i (1 - g^\infty) + \sum_{j=1}^{I} \Delta T^\infty_j \frac{\partial P^\infty_j}{\partial V(i)} \right\},
$$

(17)

Thus, summing all these equations for $i = 0, \ldots, I$, we get:

$$
\frac{\tau^\infty}{1 - \tau^\infty} = \frac{1}{\varepsilon^\infty} \cdot \frac{1}{a} \left\{ (1 - g^\infty) + \sum_{j=1}^{I} \Delta T^\infty_j \sum_{i=1}^{I} \frac{\partial P^\infty_j}{\partial V(i)} \right\} = \frac{1}{\varepsilon^\infty} \cdot \frac{1}{a} \cdot (1 - g^\infty)
$$

where the last equality is obtained using (15). This result and (17) imply that, for all $i$,

$$
\sum_{j=1}^{I} \Delta T^\infty_j \frac{\partial P^\infty_j}{\partial V(i)} = 0
$$

Assuming that the matrix $(\partial P^\infty_j / \partial V(i))_{i=1,\ldots,I; j=1,\ldots,I}$ is non singular, we obtain immediately that $\Delta T^\infty_j = 0$ for all $j$.

This result shows that the zero-tax result for spouses with high income husbands carries over to more general situations than the simple model presented in Section 3. This result is fundamentally driven by the fact that couples with very high primary earner’s income have the same marginal welfare weights (as the secondary earner’s contribution to income becomes negligible). As a result, conditional on very high earnings for the primary earner, redistributing from couples with high ability secondary earners toward couples with lower ability secondary earners has no social welfare value and therefore it is not desirable to impose a tax on secondary earners asymptotically.
4.2 Continuous Model for Secondary Earner

Instead of specifying a general discrete model for the secondary earner labor supply response, we can use a classical intensive labor supply model. In that case, the primary and secondary earner are modelled in a symmetric way as follows. There is a distribution of earnings abilities \((n_p, n_s)\) over the population of couples with density \(f(n_p, n_s)\) on the domain \(D\) and utility functions:

\[
u(c, z_p, z_s) = c - n_p h_p(z_p/n_p) - n_s h_s(z_s/n_s)\]

and \(c = z_p + z_s - T(z_p, z_s)\). This is a two-dimensional screening problem. There is a small literature in optimal tax theory considering those types of multi-dimensional screening models starting with Mirrlees (1976, 1986). There is a larger literature on multi-dimensional screening in nonlinear pricing theory (see McAfee and McMillan, 1988, Wilson, 1993, Armstrong, 1996, Rochet and Chone, 1998, and Rochet and Stole, 2002 for a recent survey).

The first order conditions for each earner imply:

\[
h'_p(z_p/n_p) = 1 - T'_p \quad \text{and} \quad h'_s(z_s/n_s) = 1 - T'_s
\]

The indirect utility is denoted by \(V(n_p, n_s)\) and satisfies (using the envelope theorem):

\[
V'_n = -h_p + (z_p/n_p)h'_p \quad \text{and} \quad V'_n = -h_s + (z_s/n_s)h'_s
\]

The objective of the government is to maximize:

\[
W = \int \int_D \delta(n_p, n_s) \Psi(V(n_p, n_s)) f(n_p, n_s) dn_p dn_s
\]

subject to the budget constraint:

\[
\int \int_D T(z_p, z_s) f(n_p, n_s) dn_p dn_s \geq R
\]

Proposition 5 The first order conditions for the optimal marginal tax rates \(T'_p\) and \(T'_s\) at ability level \((n_p, n_s)\) can be written as follows,

\[
\frac{T'_p}{1 - T'_p} = \frac{1}{\varepsilon_p} \cdot \frac{1}{n_p f(n_p, n_s)} \cdot \mu_p,
\]

\[
\frac{T'_s}{1 - T'_s} = \frac{1}{\varepsilon_s} \cdot \frac{1}{n_s f(n_p, n_s)} \cdot \mu_s,
\]
where \( \mu_p \) and \( \mu_s \) are multipliers satisfying the transversality conditions \( \mu_p(n_p, n_s) = \mu_p(\bar{n}_p, n_s) = 0 \) for all \( n_s \) and \( \mu_p(n_p, \bar{n}_s) = \mu_p(n_p, n_s) = 0 \) for all \( n_p \) and the divergence equation:

\[
\frac{\partial \mu_p}{\partial n_p} + \frac{\partial \mu_s}{\partial n_s} = [g(n_p, n_s) - 1] \cdot f(n_p, n_s),
\]

(20)

where \( g(n_p, n_s) \) is the marginal welfare weight for couples with ability \((n_p, n_s)\).

At the optimum, the following equation has to be satisfied everywhere:

\[
\frac{z_p}{n_p^2} \frac{\partial z_p}{\partial n_s} \cdot h_p'' \left( \frac{z_p}{n_p} \right) = \frac{z_s}{n_s^2} \frac{\partial z_s}{\partial n_s} \cdot h_s'' \left( \frac{z_s}{n_s} \right).
\]

(21)

The formal proof of the proposition is in appendix. The formulas are obtained from the first order conditions of the Hamiltonian. The divergence equation (20) has many solutions satisfying the boundary transversality conditions.\(^{10}\) Equation (21), which follows from the fact that the second derivative of indirect utility \( V(n_p, n_s) \) has to be symmetric, is an additional condition that has to be met and makes the optimum solution unique generically.

The optimal marginal tax rate formulas can be obtained heuristically as follows. Consider a tax reform increasing by \( dT \) the tax for couples \((n'_p, n'_s)\) above \((n_p, n_s)\) (that is, such that \( n'_p > n_p \) and \( n'_s > n_s \)). This change can be obtained by increasing the marginal tax rate on primary earners in a small interval \([n_p, n_p + dn_p]\) with spouses with ability \( n'_s \) above \( n_s \). Symmetrically, the marginal tax rate on secondary earners in a small interval \([n_p, n_p + dn_p]\) with spouses with ability \( n'_p \) above \( n_p \) is also increased. The reform is illustrated on Figure 4.

The reform therefore leads to a mechanical increase in tax revenue and a reduction in welfare for all couples in the shaded area. The net effect is:

\[
dT \int_{n_p}^{\bar{n}_p} \int_{n_s}^{\bar{n}_s} [1 - g(n'_p, n'_s)] \cdot f(n'_p, n'_s) \cdot dn'_p \cdot dn'_s
\]

In addition, there will be a labor supply response for individuals in the south and west borders of the shaded area due to changed marginal tax rates. The net loss of tax revenue is:

\[
dT \int_{n_s}^{\bar{n}_s} \varepsilon_p \frac{T_p}{1 - T_p} n_p f(n_p, n'_s) \cdot dn'_s + dT \int_{n_p}^{\bar{n}_p} \varepsilon_s \frac{T_s}{1 - T_s} n_s f(n'_p, n_s) \cdot dn'_p
\]

\(^{10}\)More precisely, if \((\mu_p, \mu_s)\) is a solution of the divergence equation, then any function \((\mu_s - \partial \phi/\partial n_s, \mu_p + \partial \phi/\partial n_p)\) where \( \phi(n_p, n_s) \) is an arbitrary scalar function will also satisfy the divergence equation.
At the optimum, those two effects need to be equal. It is straightforward to check that the resulting equation is equivalent to equations (18), (19), and (20) of the Proposition.

5 Numerical Simulations

TO BE DONE

6 Conclusion

TO BE WRITTEN
Appendix (incomplete)

Proof of Proposition 1

The government maximizes

\[ W = \int_{\bar{n}}^{n} \left\{ \int_{0}^{V(1) - V(0)} \Psi(V(n, 1) - q)p(q|n)dq + \int_{V(1) - V(0)}^{\infty} \Psi(V(n, 0))p(q|n)dq \right\} f(n)dn, \]

subject to the budget constraint

\[ \int_{\bar{n}}^{n} \int_{0}^{V(1) - V(0)} [z_p(1) + w - nh_p(z_p(1)/n) - V(n, 1)]p(q|n)f(q)dqdn + \]

\[ \int_{\bar{n}}^{n} \int_{V(1) - V(0)}^{\infty} [z_p(0) - nh_p(z_p(0)/n) - V(n, 0)]p(q|n)f(q)dqdn \geq E, \]

and the constraints arising from the couples utility maximization:

\[ V'(n, 0) = -h_p(z_p(0)/n) + (z_p(0)/n)h'_p(z_p(0)/n), \]

\[ V'(n, 1) = -h_p(z_p(1)/n) + (z_p(1)/n)h'_p(z_p(1)/n). \]

Let us denote by \( \lambda, \mu(n, 0), \text{ and } \mu(n, 1) \) the three multipliers associated. The transversality conditions are \( \mu(n, 0) = \mu(n, 1) = \mu(\bar{n}, 0) = \mu(\bar{n}, 1) = 0 \). The first order conditions with respect to \( z_p(n, 0) \) and \( z_p(n, 1) \) are

\[ \mu(n, 0) \cdot \frac{z_p(0)}{n^2}h''_p + \lambda \cdot (1 - h'_p) \cdot (1 - P(q|n)) \cdot f(n) = 0, \]

\[ \mu(n, 1) \cdot \frac{z_p(1)}{n^2}h''_p + \lambda \cdot (1 - h'_p) \cdot P(q|n) \cdot f(n) = 0. \]

The first order conditions with respect to \( V(n, 0) \) and \( V(n, 1) \) are

\[ -\frac{\partial \mu(n, 0)}{\partial n} = \int_{V(1) - V(0)}^{\infty} \Psi'(V(n, 0))p(q|n)f(n)dq - \lambda(1 - P(q|n))f(n) - \lambda[T(1) - T(0)]p(q|n)f(n), \]
\[-\frac{\partial \mu(n, 1)}{\partial n} = \int_0^{V(1)-V(0)} \Psi'(V(n, 1)-q)p(q|n)f(n)\,dq - \lambda P(q|n)f(n) + \lambda[T(1)-T(0)]p(q|n)f(n),\]

Introducing the social marginal welfare weights

\[g(n, 0) = \frac{\int_0^{V(1)-V(0)} \Psi'(V(n, 0))p(q|n)f(n)\,dq}{\lambda \cdot (1 - P(q|n))f(n)},\]
\[g(n, 1) = \frac{\int_0^{V(1)-V(0)} \Psi'(V(n, 1) - q)p(q|n)f(n)\,dq}{\lambda \cdot P(q|n)f(n)},\]

we can integrate those two equations using the upper transversality conditions and obtain:

\[\frac{\mu(n, 0)}{\lambda} = \int_n^{\hat{n}} \left\{ [g(n', 0) - 1](1 - P(q|n'))f(n') - [T(1) - T(0)]p(q|n')f(n') \right\} \,dn',\]
\[\frac{\mu(n, 1)}{\lambda} = \int_n^{\hat{n}} \left\{ [g(n', 0) - 1]P(q|n')f(n') + [T(1) - T(0)]p(q|n')f(n') \right\} \,dn'.\]

Plugging these two equations into the first order conditions for \(z_p(0)\) and \(z_p(1)\), we obtain:

\[(1-h'_p)(1-P(q|n))f(n)n = \frac{z_{p}(0)}{n}h'_p \int_n^{\hat{n}} \left\{ [1 - g(n', 0)](1 - P(q|n'))f(n') + [T(1) - T(0)]p(q|n')f(n') \right\} \,dn',\]
\[(1-h'_p)P(q|n)f(n)n = \frac{z_{p}(1)}{n}h'_p \int_n^{\hat{n}} \left\{ [1 - g(n', 1)]P(q|n')f(n') - [T(1) - T(0)]p(q|n')f(n') \right\} \,dn',\]

Using the fact that \(T'_p = 1 - h'_p\) and the definition of the labor supply intensive elasticity (6), we obtain the expressions (9) and (10) in Proposition 1.

Note that the bottom transversality conditions imply

\[
\int_n^{\hat{n}} \left\{ [g(n', 0) - 1](1 - P(q|n'))f(n') - [T(1) - T(0)]p(q|n')f(n') \right\} \,dn' = 0,
\]
\[
\int_n^{\hat{n}} \left\{ [g(n', 1) - 1]P(q|n')f(n') + [T(1) - T(0)]p(q|n')f(n') \right\} \,dn' = 0.
\]

**Proof of Proposition 2**

**TO BE COMPLETED**
References


Figure 1

The diagram illustrates the relationship between ability and tax paid. The x-axis represents ability, and the y-axis represents tax paid. The line shows two slopes: slope $\tau$ at $n$ and slope $\tau + dr$ at $n + dn$. The diagram suggests that as ability increases, tax paid also increases.
Figure 2

Tax paid

\[
\begin{align*}
\text{Slope } \tau + d\tau & \\
\text{Slope } \tau & \\
\text{Ability} & \\
\end{align*}
\]

\[
\begin{align*}
n & \quad n + dn \\
\text{working spouse} & \\
\text{non-working spouse} & \\
\end{align*}
\]
Figure 3

Tax paid vs. Ability

- Working spouse
- Non-working spouse

\[ T(1) - T(0) \text{ constant} \]